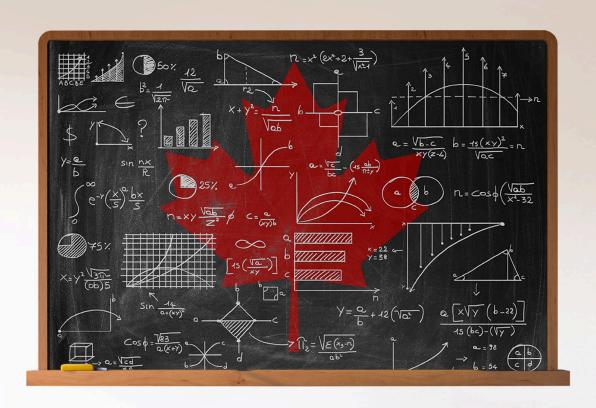


MATH PERFORMANCE IN CANADA

Derek J. Allison and Vincent Geloso





Contents

```
Executive Summary / i
Introduction / 1
International and National Assessments / 2
Provincial Assessments / 20
Conclusion / 35
References / 27
Appendix: Overview of provincial mathematics assessments
with selected trend indicators / 40
      About the Authors / 44
      Acknowledgments / 45
      Publishing Information / 46
      Supporting the Fraser Institute / 47
      Purpose, Funding, and Independence / 47
      About the Fraser Institute / 48
      Peer review—validating the accuracy of our research / 48
      Editorial Advisory Board / 49
```

Executive Summary

How are Canadian students performing in the strategically important subject of mathematics? Any useful answer depends on the measures consulted and the standards against which results are compared. Given the exclusive authority over education assigned to Canada's provincial legislatures, attention must also be given to the relative performance of provincial as well as other national systems. This study reviews the available test results in search of answers to these questions.

Initial and particular attention is given to results from the Programme for International Student Assessment (PISA). Conducted every three years, this is the most extensive and widely accepted measure of academic proficiency among lower secondary school students around the world. Results over the 2003 to 2018 period show a steady decline in Canada's math scores. Whereas Canada had the fifth highest score among the 37 countries participating in 2003, it occupied twelfth place among the 78 participating jurisdictions in the most recent 2018 assessment. Even so, Canada had the second highest 2018 math score among G7 countries after Japan, and Canadian math scores have remained significantly higher than the OECD average over this fifteen-year period. Internally, PISA scores declined in every province. Declines were steepest in Manitoba, Alberta, and British Columbia respectively, and least severe in Prince Edward Island and Ontario, with the small drop in Quebec's scores resulting in an essentially flat profile. Against the broader decline, Quebec's relatively stable scores gave it a growing lead over all other provinces.

Canada's participation in the Trends in International Mathematics and Science Study (TIMSS), the only other international assessment, is limited to students in Alberta, Ontario and Quebec. Assessments are conducted every four years. Results over the 2007 to 2015 period mirrored the PISA findings, with Grade 4 students in Quebec increasingly outperforming those in Ontario, who outperformed those in Alberta. Alberta did not participate in the Grade 8 assessment, where Quebec again outperformed Ontario. While the limited participation of provinces is unfortunate, Ontario and Quebec account for 63 percent of Canadian school enrolment, with Alberta bringing the total to three-quarters of total enrolments. Internationally, Quebec placed sixth in the 2015 TIMSS rankings, Ontario eleventh.

Interestingly, while trends from the Pan-Canadian Assessment of Science, Reading and Mathematics (PCAP), our only national achievement measure, do not agree with the PISA and TIMSS findings, the relative performance of the provinces is similar. Assessments are performed on samples of Grade 8 students every three years. Over the 2010 to 2016 period, math scores improved in all provinces except Ontario, where they were flat, and Saskatchewan, where they were hump-shaped. The greatest increases were in Prince Edward Island, Quebec, Nova Scotia, and New Brunswick respectively. Quebec had the highest average scores of all provinces in all three assessments, followed by

Ontario and Alberta, conforming to the three top-flight performers in the international assessments.

Each province has its own unique assessment system, differing in design and with regard to grade levels assessed so that results from these measures cannot be compared. Trends within provincial scores were estimated through analyses of reported results in nine provinces. Findings in each of the highest performing PISA and PCAP provinces (Alberta, Ontario and Quebec) identified one or more positive mark trends in secondary or middle school assessments, while most other provinces had flat or negative internal score trajectories.

The study concludes with a closer look at math assessments in the four provinces with the largest school systems. Differences in the scores of students in the French and English language systems in Quebec and Ontario are examined, as are scores in Ontario's soon to be discontinued Academic and Applied Grade 9 courses. Comparative results from Alberta's Grade 6 and 9 assessments are considered, together with trends in results from the province's Grade 12 math examinations. British Columbia's criterion-referenced results are compared to average scores to highlight interpretive differences, and persistent differences in the scores of students attending public and independent schools are considered.

On balance, Canadian math scores are in decline overall, while the relative performance of the provinces has remained stable, with Quebec consistently outperforming all other provinces on all available comparative measures, and Ontario and Alberta occupying second and third place respectively. Results from the Pan-Canadian assessments paint a partially contradictory picture, showing increasing scores in all provinces except Ontario and Saskatchewan, but with a similar pattern of relative performance across the provinces. The differences in score trends are likely attributable to a focus on measuring the achievement of common Canadian curriculum expectations in the PCAP assessment, rather than the broader conception of mathematical proficiency underlying the international assessments. Given that the provinces measure math performance against their own curriculum standards, consideration might usefully be given to adopting a broader conceptual framework for the Pan-Canadian assessment similar to those used in international assessments.

Introduction

Mathematics holds a position of strategic importance in the school curriculum, increasingly so in our ever more digital world. Yet there have been worrying signs of declining math performance by Canadian students, with educators, parents, and politicians expressing concern (e.g., Canadian Mathematical Society, 2010; Hammer and Alphonso, 2018; Logan, 2014; Roshowy, 2019). This has encouraged calls for remedial measures including curriculum changes and enhanced teacher training (Stokke, 2015). Ontario's Minister of Education recently acknowledged such concerns when announcing a \$200 million, four-year overhaul of math instruction in the province, which will include a revised elementary math curriculum (Ontario, 2019). Even so, accounts of students' math performance typically concentrate on results from a single province, as was the case in the Ontario announcement, or on results from international assessments, particularly the Programme for International Student Assessment (PISA). This, together with differences in test design and the grade levels at which students were tested, makes it difficult to discern overall trends.

This study seeks to address these difficulties by drawing on available measures to construct a comprehensive comparative account of math performance in Canada, in search of a consensus on discernable trends. As such, this is a descriptive work intended to help clarify the state of math learning in Canada. We take the Programme for International Student Assessment (PISA) over the period from 2003 to 2018 as providing the most reliable benchmark data. These results are used to place Canadian performance in an international context, and to compare results from the provinces. We then consider results from the two other available measures providing provincial comparisons. Finally, we offer an overview of provincial math assessments, concluding with more detailed reviews of results in the most populous provinces of Ontario, Quebec, Alberta, and British Columbia. Because each province designs and administers its own math assessments, results are not comparable. Even so, internal trends in individual provinces help solidify general conclusions, while the overview of the provincial assessment policies illustrates the wide range of different measures in use.

What emerges from the sources considered is a relatively consistent portrait. Most results paint a picture of declining math performance for Canada as a whole and in the provinces, with the notable exception of Quebec. Only results from the Pan-Canadian Assessment Program (PCAP) yield improving scores, but the design of this assessment appears less than fully suited for interprovincial comparisons. There is nonetheless general agreement on relative provincial performance. Quebec has performed consistently above the Canadian average over the fifteen years considered; Alberta, British Columbia, and Ontario have fared relatively well, placing around the Canadian average, although B.C. scores have been slipping in more recent assessments. The Atlantic provinces fare less well, and Manitoba and Saskatchewan even less so.

International and National Assessments

Programme for International Student Assessment (PISA)

The most widely accepted assessment of student and system performance is conducted by the Programme for International Student Assessment (PISA) under the aegis of the Organisation for Economic Co-operation and Development (OECD). Administered at three-year intervals since 2000 in the core academic domains of reading, mathematics, and science, it provides a comprehensive set of comparable measurements for all OECD members and participating partner countries and economies (Cordero et al., 2018; Zamarro et al., 2016). Comparable measures for some sub-national jurisdictions, including the Canadian provinces, are also available. The PISA assessments measure student performance at 15 years of age so that almost all participating Canadian students are in Grade 10. [1] The presumption is that differences in test scores reflect differences in content knowledge and procedural ability.

Sophisticated random sampling is used to maximize coverage and minimize bias in selecting participants. [2] First, a stratified random sample of schools is drawn in each participating jurisdiction. [3] Second, a representative sample of students within each school is selected (OECD, 2019a). The tests are conducted within the same time window to minimize differences related to the amount of instruction received. Additional procedures are implemented to account for schools that exit the survey or fail to provide acceptable response rates. Test scores are standardized and scaled to a mean of 500 with a standard deviation of 100.

Because the assessments are derived from samples of national and sub-national populations, all reported scores are estimates of student performance rather than exact measures. Both the samples and the tests themselves are designed to reduce and manage the associated uncertainty. All results have accompanying standard error statistics. This allows the margin of error to be determined by calculating confidence intervals around the estimated score. These confidence intervals establish high and low boundaries that have a high probability of bracketing the true population value. We follow the PISA convention of reporting statistical significance based on 95 percent confidence levels (p. < .05). When estimated scores fall within these confidence levels there is less than a five percent chance that the true population value falls outside the confidence interval. When comparing scores between jurisdictions or over time, overlapping confidence intervals are interpreted as implying results are statistically similar as there is a

^{[1] 87.7} percent for Canada as a whole in the 2018 assessment (OECD, 2019a: Annex A2, Table I.A2.8).

^[2] There are some exclusions that are allowed, but these are small in number.

^[3] The stratification is designed to provide proper subnational representations (e.g., states, provinces, rural, urban) and to reflect institutional makeup of student population (e.g., type of school—although proportions of independent (private) schools are not always accurately represented).

95 percent chance that there is no actual difference between the population values of the scores being compared.

Sampling errors increase as sample sizes decrease, so that there is greater uncertainty with smaller samples. Larger samples as usually found at the national level normally have smaller standard errors and thus tighter confidence intervals than the characteristically smaller samples at sub-national levels, particularly in the smaller Canadian provinces. The sampling errors associated with average performance estimates at the national level are typically on the order of two to three PISA score points (OECD, 2019a: 45). Sampling errors for the Canadian provinces are larger. [4]

Tests and test items change from assessment to assessment introducing further uncertainty when comparing results over time. This is addressed through various techniques which quantify the comparative measurement uncertainty of the tests used in different years. This uncertainty is expressed as the link error between, for example, math scores in 2003 and 2006, or any other assessment years. Link errors pertain to assessments and are independent of sample sizes. One of the three subject domains receives more detailed attention every three assessment cycles which, in addition to allowing for more detailed analysis, provides the additional data used to calculate the link errors. For this reason, time comparisons of subject scores can only be properly made with reference to the first year in which the subject was a major domain. The first year in which math was the major domain was 2003 so this is the base year for time comparisons of math performance. Link errors are the most significant sources of uncertainly when comparing PISA scores over time. In the discussions that follow the appropriate link errors have been incorporated into the calculation of the confidence levels used to determine the statistical significance of differences between math scores.

The PISA survey is a complex study with multiple layers of assessments. Considerable attention is given to maximizing the comparability of results from cycle to cycle so as to ensure apples are compared with apples. These efforts are subject to intense methodological discussions. The reading domain is subject to debates between experts over language differences that may make comparisons difficult (Goldstein, 2017). There are also issues that are known to exist with regard to how administrative directives are followed and the tests are written (Grisay et al., 2007), or how participating students may be selected so as to minimize the effect of skimming from the top or bottom of in-school performance distributions. Care is also taken to account for how cultural differences may induce different interpretations which could cause inaccurate comparisons between countries.

These problems are more limited in the case of math performance given the greater universality of mathematical concepts. Indeed, assessments of methodological points of contention suggest very little differences in national rankings when adjustments intended to compensate for such differences are made to the math scores (Jerrim et al., 2018). Most of the methodological concerns regarding performance assessment in mathematics relate

^[4] The mean sampling error for math scores from the Canadian provinces ranged from 2.88 in 2003 to 7.21 in 2015.

to sampling procedures and the comparability of results from widely differing countries. Within Canada there are fewer variations that could cause errors in measurement and we can feel more confident with regard to the results.

PISA test data are most often used in the literature when comparing student and system performance largely because of the considerable effort taken to arrive at apples to apples comparisons (Cordero et al., 2018). While not flawless, PISA clearly constitutes the best available measure. As such, throughout this paper we benchmark most findings against the full range of PISA math results from 2003 to 2018.

Canada's PISA results in international context

Canada's PISA math scores have been declining. Canada had an estimated average math score of 532 in the 2003 PISA assessment, which was the fifth highest among the 37 countries participating in that baseline assessment. In that year Canada's math score was statistically lower than top ranked Hong Kong (550), Finland (544) and Korea (542), and statistically similar to the Netherlands (538), Japan (534) and Belgium (530). In the most recent 2018 assessment, Canada's math score had fallen 20 score points to 512, placing Canada in twelfth place among the 78 participating jurisdictions. The five highest scoring jurisdictions in 2018 were all non-OECD Asian economies. [5] Canada placed seventh among OECD members, with a statistically lower score than Japan (527), Korea (526), Estonia (523), and the Netherlands (519). Even so, Canada had the second highest 2018 math score among G7 countries after Japan.

Canada's high rankings among G7 countries are noteworthy accomplishments. Yet Canada was one of only five countries [6] experiencing a decline of 20 score points or more in math from 2003 to 2018. Table 1 compares math scores and various trend measures for Canada, the G7, and other selected OECD countries. Finland, once celebrated for exemplary PISA performance, recorded the greatest score decline among OECD countries, experiencing a loss of 37 score points from 2003 to 2018, which represents an average loss of 6.2 points each assessment for an overall percentage decline of 6.8 points over the 2003/18 period, and 2.3 percentage points over the last three assessments. Overall, Finland's scores declined consistently between 2006 and 2018, conforming to what PISA classifies as an "Increasingly negative" score trajectory (OECD, 2019a: 134). Canada's "Steadily negative" trajectory describes a less severe decline with an average loss of 3.3 percentage points per assessment, and a 3.8 percent decline over the 2003 to 2018 period, flattening to a 1.2 percent decline over 2012 to 2018. Overall, Canada experienced a sharp, statistically significant drop in PISA math scores between 2009 and 2012, followed by a continuing but shallower decrease thereafter.

^[5] These were the provinces/municipalities of Beijing, Shanghai, Jiangsu and Zhejiang, collectively referred to as B-S-J-Z (China) with a score of 591, Singapore (569), Macao (China) (558), Hong Kong (China) (551) and Chinese Taipei [Taiwan] (531) (OECD, 2019a: Table I.B1.11).

^[6] The remaining four were Finland (37 points), Australia (33 points), New Zealand (29 points), and Belgium (21 points). As shown in table 1, Canada's score declined 20 points over the 2003 to 2018 period.

Table 1: Estimated average PISA math scores for Canada, G7, and selected OECD countries

	Assessment year							Score	differenc	Trajectory	
Country	2003	2006	2009	2012	2015	2018	Total	Avg	6cy%∆	Зсу%∆	
Finland	544	548	541	519	511	507	-37	-6.2	-6.8%	-2.3%	Increasingly negative
Canada	532	527	527	518	516	512	-20	-3.3	-3.8%	-1.2%	Steadily negative
France	511	496	497	495	493	495	-16	-2.7	-3.1%	0.0%	Negative but flattening
Japan	534	523	529	536	532	527	-7	-1.2	-1.3%	-1.7%	Flat
OECD avg	499	497	499	496	491	494	-5	-0.8	-1.0%	-0.4%	Flat
USA	483	474	487	481	470	478	-5	-0.8	-1.0%	-0.6%	Flat
Germany	503	504	513	514	506	500	-3	-0.5	-0.6%	-2.7%	Hump-shaped
UK	DNP	495	492	494	492	502	7	1.2	1.4%	1.6%	U-shaped
Estonia	DNP	515	512	521	520	523	8	1.3	1.6%	0.4%	Steadily positive
Italy	466	462	483	485	490	487	21	3.5	4.5%	0.4%	Positive but flattening
Portugal	466	466	487	487	492	492	26	4.3	5.6%	1.0%	Positive but flattening

Notes: Countries ranked by increasing total score difference 2003-18. DNP = Did not participate. Scores in bold indicate statistically significant difference (p. <.05) from 2003 baseline. 6cy% Δ is percentage change in scores over all six assessment cycles (five for UK and Estonia); 3cy% Δ is percentage change in scores over the last three assessment cycles (2012-18). Trajectory descriptions as assigned by PISA (OECD, 2019a: 133).

Source: OECD, 2019a: Table I.B1.11; authors' calculations.

Scores for Japan and the USA clustered around the OECD average, with score trajectories for all three categorized as "Flat". Both countries and the OECD average experienced small, non-significant score declines overall, with longer- and shorter-term percentage decreases around 1 percent. Trajectories for Germany and the UK were more variable, with modest changes overall. The remaining three countries in table 1 all recorded increasing math scores. Estonia achieved a modest but significant 1.6 percent gain over five assessments along a "Steadily positive" trajectory. Italy was the only G7 country to record significantly increasing math scores over the six PISA assessments. Portugal had the greatest gain among OECD countries participating in all six math assessments, with a notable increase of 26 score points, amounting to a 5.6 percentage change over all assessments, flattening to a 1.0 percentage increase over the last three assessments, for a "Positive, but flattening" trajectory.

Figure 1 places Canada's declining math scores in the visual context of score changes in selected countries. The 'T' shaped error bars extending from the plotted values show the confidence intervals above and below the estimated scores. Canada and Japan had the highest estimated average math scores among G7 countries over the six 2003 to 2018 assessments. As shown by the overlapping error bars, these scores were statistically similar in the first three assessments, but the differences became statistically significant after Canada's scores dropped sharply in 2012. Math scores for the United States remained statistically flat but, while the USA had the lowest math scores among G7 countries over

540 Japan 530 520 Estonia 510 Canada **OECD** Average 500 490 USA 480 470 Portugal 460 2003 2006 2009 2015 2012 2018

Figure 1: PISA estimated math scores with confidence intervals, Canada and selected OECD countries, 2003 to 2018

Note: Lines have been slightly offset from each other due to overlap of error bars. Source: OECD, 2019a: Table I.B1.11; authors' calculations.

the 2012 to 2018 period, Canada's declining performance narrowed the score point difference between the two counties from 46 in 2015 to 34 in 2018. Even so, scores for Japan and Canada remained significantly higher than the OECD average throughout this period, while those for the USA remained significantly lower. The positive performances of Estonia and Portugal are evident in figure 1, Portugal moving from an average score estimate statistically lower than the USA in 2003 to statistically higher scores in 2015 and 2018 to match the OECD average. Similarly, Estonia moved from a statistically lower average score than Canada in 2009, to a statistically higher score estimate in 2018.

In summary, table 1 and figure 1 show a statistically significant decline in Canada's international math scores over a fifteen-year period, while also showing improvements in selected comparable countries. Canadian math scores have nonetheless remained significantly higher than the OECD average over this period, and significantly higher than US scores, although US scores remained stable while Canada's declined. On balance, while Canadian math scores have declined, Canada has maintained a commendably high place among OECD and G7 countries. Yet, as noted earlier, these predominantly Western countries are overshadowed by the five Asian economies named in footnote 5, each of which significantly and substantially outperformed Canada in the 2018 results.

Provincial trends

Canadian students recorded lower average PISA math scores on all of the assessments after 2003, except for 2009, when there was no change from 2006. In spite of this steady decline, Canada's average 2018 math score of 512 remains significantly higher than the OECD average of 489 by a comfortable margin. [7]

Even so, Canada's falling math scores hide some more pronounced provincial declines, as summarized in **table 2** and **figure 2**. In both data displays, provinces are ordered by decreasing 2018 scores so that Quebec, with the highest 2018 score of 532, appears first and Manitoba, with a 2018 score of 482, last. The first four provinces shown (QC, ON, AB, BC) had the highest scores in all six assessments, but occupied varying places. Together, these provinces educate by far the great majority of Canadian students (87.4 percent). [8]

Table 2: Estimated average PISA math scores with trend indicators, Canada and provinces, 2003 to 2018

	Assessment year						Score difference			Trajectory	
Jurisdiction	2003	2006	2009	2012	2015	2018	Total	Avg	6cy%∆	3су%∆	
QC	536	540	543	536	544	532	-4	-0.7	-0.7%	-0.7%	Flat
ON	530	526	526	514	509	513	-17	-2.8	-3.2%	-0.2%	Negative but flattening
Canada	532	527	527	518	516	512	-20	-3.3	-3.8%	-1.2%	Steadily negative
AB	549	530	529	517	511	511	-38	-6.3	-6.9%	-1.2%	Negative but flattening
ВС	538	523	523	522	522	504	-34	-5.7	-6.3%	-3.4%	Increasingly negative
NS	515	506	512	497	497	494	-21	-3.5	-4.1%	-0.6%	Steadily negative
NB	511	506	504	502	493	491	-20	-3.3	-3.9%	-2.2%	Steadily negative
NL	517	507	503	490	486	488	-29	-4.8	-5.6%	-0.4%	Negative but flattening
PE	500	501	487	479	499	487	-13	-2.2	-2.6%	1.7%	U-shaped
SK	516	507	506	506	484	485	-31	-5.2	-6.0%	-4.2%	Increasingly negative
MB	528	521	501	492	489	482	-46	-7.7	-8.7%	-2.0%	Steadily negative

Notes: Jurisdictions ranked by highest to lowest 2018 scores. Scores in bold indicate statistically significant difference (p. <.05) from 2003 baseline. 6cy% Δ is percentage change in scores over all six assessment cycles; 3cy% Δ is percentage change in scores over the last three assessment cycles (2012-18). Trajectory descriptions from the nine variants identified by PISA (OECD, 2019a: 133) as assigned by authors...

Source: OECD, 2019b: Tablel.B2.10; authors' calculations.

^[7] In 2003, the average OECD score stood at 499 which meant that Canada was 6.4 percent above the OECD average. Relative to the OECD 2018 mean score of 494, Canada's advantage had fallen to 3.6 percent. [8] Students attending all public and independent schools and programs in 2017/18 (Statistics Canada, 2020). Despite year-to-year variation in enrolments, the relative shares of provincial enrolments have been stable since 2000 with the exception of Alberta replacing British Columbia in third place over the 2006/07 to 2007/08 period. Respective 2017/18 proportions for the largest four provinces were: ON 38.9 percent, QC 24.2 percent, AB 12.7 percent, and B.C. 11.7 percent.

560 **2**003 **2**006 **2**009 **2**012 **2**015 **2**018 550 540 530 520 510 500 490 480 470 460 BC NS NB QC ON Canada ΑB NL

Figure 2: Estimated average scores and confidence intervals for PISA math results, Canada and the provinces, 2003 to 2018

Notes: Jurisdictions ranked from highest to lowest 2018 scores. The linkage error is incorporated into the standard error for 2006 and subsequent years. For some provinces, the standard errors differ from those in previous PISA reports due to OECD methodological changes. Source: CMEC, 2018: Table B.3.15a; authors' calculations.

As shown in the two data displays, Alberta had the highest score of 549 in the 2003 assessment, British Columbia ranked second with 538, Quebec third with 536, and Ontario fourth with 530, slightly below the Canadian average of 532. As shown by the overlapping confidence interval error bars in figure 2, Alberta's 2003 score was statistically similar to British Columbia and Quebec, but significantly higher than all other provinces, including Ontario. By maintaining a relatively flat trajectory over successive assessments, Quebec assumed and then maintained the highest score from 2006 onward with scores significantly above the national score. In contrast, scores for Alberta and B.C. declined steeply. After a significant decline over the 2003 to 2012 period, Ontario's scores stabilized so that, while Ontario's 2018 score of 513 is significantly below its 2003 score, Ontario had the second highest score of all provinces in those most recent results. With the exception of Manitoba, 2003 scores in the remaining provinces were significantly lower than the Canadian score and continued to decline thereafter at variable rates. Manitoba ranked fifth among the provinces in 2003 with a score statistically similar to Quebec, Ontario, and Canada overall, but Manitoba's score then declined steadily and steeply.

Table 2 shows estimated average scores and selected trend indicators for Canada and each province, with entries ordered by the estimated 2018 average scores as in figure 2. Quebec's essentially negligible decline of 4 score points for an average 0.7 percent decrease over the 2003 to 2018 period yields a flat score trajectory over the six assessments with no statistically significant change from the 2003 baseline score. With this

exception, the overall pattern is one of decline. As noted, the 2018 scores for all provinces were lower than their 2003 baseline scores, significantly so for all but Quebec and Prince Edward Island. Prince Edward Island had the second smallest decline of 13 score points over the six assessments. Yet, P.E.I.'s U-shaped score trajectory includes scores that were significantly below the 2003 baseline in 2009 and 2012, until achieving a marginal but non-significant gain over the other Atlantic provinces in 2015. Still, P.E.I.'s scores were the lowest in Canada until the 2015 assessment. In 2017/18, the province enrolled only 0.4 percent of all Canadian students (Statistics Canada, 2020). The 2018 scores of the other three Atlantic provinces were statistically similar to P.E.I. over the 2012 to 2018 period, as were the scores for Saskatchewan and Manitoba in 2015 and 2018. Taken together, these six provinces enrolled 12.6 percent of Canadian students in 2018. Score trajectories in these provinces varied over the six assessments. New Brunswick and Nova Scotia experienced relatively modest but steady and statistically significant declines similar to Canada overall. Scores for Newfoundland and Labrador declined more substantially over the first three assessments, before flattening over the last three assessments.

With a percentage decrease of six percent or more over the 2003 to 2018 period, each of the Western provinces experienced the greatest overall declines in math scores. Alberta's loss of 38 score points was concentrated in the 2003 to 2012 period with a flattening trajectory after 2012. As shown in figure 2, scores in British Columbia followed more of a stepped trajectory, with a stable period between 2006 and 2015 after a drop from 2003 to 2006, culminating in a substantial drop in 2018. Saskatchewan's score trajectory follows a similar stepped trajectory, but with a steep drop in 2015. Manitoba's leading decrease of 46 score points for a decline of 8.7 percent was distributed relatively evenly over all six assessments.

Overall, the national-level score is prevented from showing a steeper decline by the contributions of Quebec, Alberta, and British Columbia in the earlier assessments, and Quebec and Ontario in more recent years. Figure 3 provides a graphical overview of the comparative magnitude of provincial declines by charting the percentage change in estimated average scores from the 2003 baseline to 2018. Prince Edward Island's volatile scores over the six assessments, as shown in table 2 and discussed earlier, suggest its current second ranked position may not be sustained into the future. Ontario's more stable score trajectory over the last three assessments implies a more solid comparative ranking. Regardless, the high scores and smaller percentage declines of Quebec and Ontario, together with their combined 63 percent share of Canadian enrolment, boost the national score. In this respect it is worth noting that Quebec ranked fifth internationally in the 2018 PISA results with a score statistically similar to Japan and significantly higher than Estonia, but significantly lower than the five Asian countries listed in footnote 5. Less spectacularly but still commendably, Ontario's 2018 score was statistically similar to Estonia but significantly higher than France, Australia, Italy, and the USA. [9]

^[9] Alberta's 2018 score was also significantly higher than these nations, although Alberta scored significantly below Estonia.

0.0% -0.7% -2.0% -2.6% -3.2% -4.0% -3.8% -3.9% -4.1% -6.0% -5.6% -6.0% -6.3% Decline not statistically significant -6.9% -8.0% -8.7% -10.0% NL BC QC PF ONCanada NR NS SK AB MR

Figure 3: Percentage change in average PISA math estimates, Canada and provinces, 2003 to 2018

Source: ????

In sum, all provinces except Quebec and Prince Edward Island suffered statistically significant declines from their 2003 baseline PISA scores, with P.E.I.'s scores recovering from statistically significant lows in 2009 and 2012. Seven provinces, enrolling three-quarters of Canadian students in total, experienced greater percentage declines than the Canadian average. Taken together, the Western provinces—the three prairie provinces and British Columbia—suffered the greatest score declines. The declines in Alberta and British Columbia are particularly notable: both provinces stood above the national score in 2003, but have since experienced substantial declines.

Trends in International Mathematics and Science Study (TIMSS)

A second data source providing interprovincial comparisons is the Trends in International Mathematics and Science Study (TIMSS), conducted by the International Association for the Evaluation of Educational Achievement (IEA). The TIMSS is similar in design to PISA as it is meant to provide international comparisons of student achievements in science and mathematics in Grades 4 and 8 every four years.

The TIMMS results are often used as a complement to the PISA results because they focus on achievements at different school grades. The two tests are practically quite similar as suggested by the high correlation between the two measures (Klieme, 2016). [10] In terms of quality, TIMSS rivals PISA. As with PISA, scores are standardized to a mean of 500 with a standard deviation of 100 to facilitate comparisons.

^[10] Klieme reports a correlation of .923 between TIMSS Grade 8 and PISA 2015 math scores for 27 comparable countries, a .931 correlation for TIMSS Grade 8 2007 and PISA 2006 (25 countries), and .944 for TIMSS Grade 8 2011 and PISA 2012 (28 countries).

Unfortunately, Canadian participation is limited. Although other provinces participated at reduced sampling levels, [11] results have only been published for Grade 4 and 8 students in Quebec and Ontario from 2003 to 2015, and for Grade 4 Alberta students from 2007 to 2015. [12] Results are not available for the other provinces. Even though Ontario and Quebec enrol almost two-thirds (63.1 percent) of Canadian school students, the incomplete data has led some international researchers to exclude Canada in their analyses of the TIMMS results (Broer et al., 2019: 21). Nevertheless, the TIMSS data that are available complement and extend the insights provided by PISA.

Table 3 shows estimated average TIMSS math scores together with selected trend indicators. Figure 4 offers graphical comparisons with error bars mapping respective confidence intervals around the plotted scores. Quebec's status as a consistent high performer is confirmed, with significantly higher Grade 4 and 8 scores in the two most recent assessments.

Table 3: Estimated average TIMSS math scores with trend indicators, participating provinces, 2003 to 2015

	,	Assessi	nent ye	ar		Score	differenc	ce	Trajectory
Jurisdiction	2003	2007	2011	2015	Total	Avg	4cy%∆	2cy%∆	
Grade 4									
Quebec	506	519	533	536	30	7.5	5.9%	0.6%	Positive, but flattening
Ontario	511	512	518	512	1	0.3	0.2%	-1.2%	Flat
Alberta	DNP	505	507	484	-21	-7.0	-4.2%	-4.5%	Increasingly negative
Grade 8									
Quebec	543	528	532	543	0	0.0	0.0%	2.1%	U-shaped
Ontario	521	517	512	522	1	0.3	0.2%	2.0%	Flat

Notes: Provinces ranked by increasing negative score point difference. Scores in bold indicate statistically significant difference (p. <.05) from 2003 baseline (2007 for Alberta). DNP = Did not participate. 4cy% Δ is 4 cycle percentage change (2003/15). 2cy% Δ is 2 cycle percentage change (2011 & 2017). Trajectory descriptions from the nine variants identified by PISA (OECD, 2019a: 133) as assigned by authors.

Source: CMEC, 2017: Tables 1.10 & 2.10.

[11] Manitoba and Newfoundland and Labrador participated in the 2015 assessments but at sampling levels insufficient for reliable comparative results. Aggregated results from all participating provinces were nonetheless used to calculate the Canadian average reported by the Council of Ministers of Education Canada (2017), but in the absence of applicable published results for all provinces these "Canadian" averages are not reported here.

[12] TIMSS data are also available for 1995 (Grade 4) and 1999 (Grades 4 and 8) but we have not included these earlier results in order to focus specifically on a time span directly comparable to the PISA results. The earlier data show a significant drop in Quebec's Grade 4 scores from 1995 to 2003 with a steady increase thereafter. A similar but less pronounced pattern appears in Quebec's Grade 8 scores with a significant decline between 1999 and 2003 and flat scores thereafter. Ontario experienced a statistically significant increase in Grade 4 scores from 1995 to 2003, remaining essentially flat in the following years. A significant gain in Ontario's Grade 8 scores between 1995 and 2003 mirrored the Grade 4 results, with most of this gain occurring between 1995 and 1999.

550 Ouebec, Grade 8 540 Quebec 530 Grade 4 Ontario, Grade 8 520 510 Ontario, Grade 4 500 Alberta, Grade 4 490 480 470 2003 2007 2011 2015

Figure 4: Estimated Grade 4 average scores and confidence intervals, TIMSS math, 2003 to 2015

Note: Lines have been slightly offset from each other due to overlap of error bars. Source: CMEC, 2017: Tables 1.10 & 2.10.

As shown in figure 4 and with reference to entries in table 3, Quebec's Grade 4 math scores increased steadily from cycle to cycle resulting in a statistically significant gain over the 2003 baseline by 2011, which is sustained until 2015, but at a reduced pace, describing a positive, but flattening trajectory. Ontario's Grade 4 scores traced a flat trajectory over the full 2003 to 2015 period, with no statistically significant gains or decreases. Alberta's Grade 4 scores followed an increasingly negative trajectory, remaining steady over the 2007 to 2011 period before a significant 23 score point drop from 2011 to 2015. In the most recent 2015 results, Quebec's average estimated score of 536 is significantly higher than Ontario's 512, which is itself significantly higher than Alberta's 484. These provinces occupied the same ranks in the 2011 TIMSS results, with Quebec significantly outscoring Ontario whose score was statistically similar to Alberta in that assessment.

Figure 4 shows Quebec significantly outperforming Ontario at the Grade 8 level in all but the 2007 assessment, when Quebec's estimated average score fell significantly below the 2003 baseline score and was statistically similar to Ontario's. Increasing scores thereafter brought Quebec's Grade 8 results back to statistical parity with the 2003 baseline. Ontario's Grade 8 scores remained flat over the 2003 to 2015 period with no statistically significant gains or losses. Under the TIMSS design, each Grade 4 student cohort is sampled again four years later. In this respect, the consecutive increases in Quebec's Grade 4 scores over adjacent assessment cycles from 2003 to 2015 broadly agree with the improving Grade 8 score over the 2007 to 2015 period, reinforcing the reliability of the results. Similarly, the statistically flat trajectory of Ontario's Grade 4 scores over the 2003 to 2015 time span is similar to the flat trajectory of the lagged Grade 8 scores.

These relative performance patterns are consistent with the PISA results for 15 year-olds, the great majority of whom are enrolled in Grade 10 courses. Comparative TIMSS Grade 8 results agree with the PISA data and are internally consistent with the TIMSS Grade 4 results. While the summative descriptive trajectories in tables 2 and 3 are not in full agreement, they are not contradictory. Quebec's PISA-based "flat" trajectory in table 2, for example, is broadly consistent with the Grade 8 TIMSS shallow "U-shaped" trajectory in table 3, with very small score differences between the baseline and terminal assessment years. Ontario has a greater score point difference in the PISA data, but the flattening of scores over the 2012 to 2015 period is compatible with the TIMSS profile.

Finally, the comparative international rankings from the TIMSS data agree with those from the PISA results, with a few interesting anomalies. In PISA 2018, Quebec ranked fifth overall after the four highest-scoring Asian countries listed in footnote 5, with a score of 532, which was statistically similar to Japan's 527. In the Grade 8 TIMSS results, Quebec was sixth with a score of 543, close to but statistically lower than Japan's 586. Ontario placed thirteenth in PISA18 with a score (513) statistically lower than Japan but considerably greater than the USA (478). In the TIMSS Grade 8 rankings, Ontario placed eleventh with a score of 522, statistically lower than Japan (586) but statistically similar the USA (518). As shown earlier in figure 1, the USA's PISA math scores have been flat and significantly lower than Canada's. In notable contrast to the pattern of TIMSS results in Canada, Grade 8 TIMSS results in the USA have been consistently lower than the USA's Grade 4 scores. In conformity with this pattern, the USA's 2015 Grade 4 score of 539 was statistically similar to Quebec's 536 and significantly higher than Ontario's 512 and Alberta's 484, even though all three provinces outperformed the USA in the Grade 10 PISA results.

Pan-Canadian Assessment Program (PCAP)

Canada has only one set of national school achievement tests: the Pan-Canadian Assessment of Science, Reading and Mathematics (PCAP), sponsored by the Council of Ministers of Education, Canada (CMEC). The PCAP is structurally similar to PISA and TIMSS with scores standardized to a mean of 500 and a standard deviation of 100. Grade 8 students in randomly selected classrooms in randomly selected schools in each province respond to subsets of a larger universe of test items which are pooled for analysis (CMEC, 2016). Test items are developed and field tested in Canada with reference to provincial curricular expectations and with the involvement of curriculum experts and teachers across the country. Assessments have been conducted every three years since 2007. One subject receives more detailed attention every nine years as the rotating major domain. This additional attention includes a wider range of test items including common "anchor items" designed to provide baseline linked results to facilitate comparisons over time (O'Grady, Fung, Servage, and Khan, 2018: 5). Such comparisons can only be properly done with reference to the first assessment in which a subject was the major domain. This was 2010 for mathematics. Consequently the following review is limited to results from the three assessments conducted in 2010, 2013, and 2016.

The architecture and administration of PCAP resembles the PISA and TIMSS studies (Volante, 2016). But whereas PISA and TIMSS are designed to assess mathematical reasoning and application in an international context, PCAP focusses directly and exclusively on Canadian needs and expectations with specific attention to Grade 8 curriculum areas common to the provinces (O'Grady and Fung, 2016: 10). Results are intended to be used primarily by provincial policy makers to monitor, assess, and improve their education systems (O'Grady, et al., 2018: 6).

Results for the three comparable years are shown in table 4 and figure 5, with provinces ranked according to their decreasing 2016 scores to facilitate comparison with the PISA results shown earlier. There is evident disagreement between the two sets of results: While the PISA scores portray variable rates and patterns of decline in all provinces, the PCAP results show variable patterns and rates of increase in all provinces.

There were statistically significant increases in PCAP math scores at the national level and in nine provinces over the three assessments. Only Ontario failed to record statistically significant increases, scores following a flat trajectory with no significant gains or losses. Results in Saskatchewan traced a hump-like trajectory with a statistically significant gain from 2010 to 2013 followed by a non-significant decline over the 2013 to 2016 period, resulting in a 2016 score that remained significantly higher than the 2010 baseline. Scores in all other provinces followed either a steadily positive or an increasingly

Table 4: Estimated average PCAP math scores with trend indicators, Canada and provinces, 2010 to 2016

	Assessment year			Score difference			Trajectory
Jurisdiction	2010	2013	2016	Total	Avg	%∆	
Quebec	515	527	541	26	8.7	5.0%	Steadily positive
Canada	500	507	511	11	3.7	2.2%	Steadily positive
Ontario	507	512	508	1	0.3	0.2%	Flat (non-significant hump)
Alberta	495	502	505	10	3.3	2.0%	Steadily positive
Prince Edward Island	460	492	503	43	14.3	9.3%	Increasingly positive
New Brunswick	478	480	498	20	6.7	4.2%	Increasingly positive
Nova Scotia	474	488	497	23	7.7	4.9%	Steadily positive
British Columbia	481	489	494	13	4.3	2.7%	Steadily positive
Newfoundland and Labrador	472	487	490	18	6.0	3.8%	Steadily positive
Saskatchewan	474	488	483	9	3.0	1.9%	Hump-shaped
Manitoba	468	471	479	11	3.7	2.4%	Steadily positive

Notes: Jurisdictions ranked by decreasing 2016 scores. Scores in bold indicate statistically significant difference (p. <05) from 2003 baseline. $\%\Delta$ is percentage change from 2010 to 2016. Trajectory descriptions from the nine variants identified by PISA (OECD, 2019a: 133) as assigned by authors...

Source: O'Grady, Fung, Serage, and Khan, 2018: Table B.13; authors' calculations.

550 **2010 2013 2016** 540 530 520 510 500 490 480 470 460 450 QC Canada ON AB PE NB NS BC NL SK MB

Figure 5: Estimated PCAP average Grade 8 math scores and 95% confidence intervals, Canada and provinces, 2010 to 2016

Source: O'Grady, Fung, Serage, and Khan, 2018: Table B.16; authors' calculations.

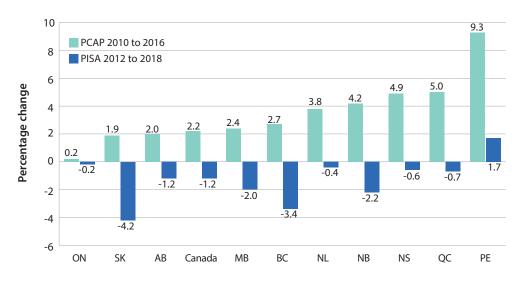
positive trajectory. The 9.3 percentage increase for Prince Edward Island is exceptional, catapulting the province from the lowest score in 2010 to the fourth highest provincial score in 2016, statistically higher than Newfoundland and Labrador, Saskatchewan, and Manitoba. Yet despite this substantial increase, Prince Edward Island's 2016 score remained 38 points below Quebec's dominating score of 541. Quebec's performance was itself exceptional with significantly higher scores than all other provinces in each assessment, as well as significant assessment-to-assessment gains within the province, resulting in a greater percentage increase than any other province except P.E.I.. Quebec's 2016 score of 541 was 33 score points higher than Ontario's next highest score of 508. Not only did Quebec perform consistently above the national result in each assessment, the score point difference between Quebec and the national average increased at an accelerating pace, from 15 points in 2010 to 20 in 2013 and 30 in 2016.

While each province except Ontario experienced statistically significant gains over their 2010 baseline results, scores for the three provinces at the lower end of the score distribution remained well below the national score in each of the three assessments. Both Newfoundland and Labrador and Saskatchewan recorded large, significant increases between 2010 and 2013, before leveling off, in the case of Newfoundland and Labrador, or declining in Saskatchewan, leaving Saskatchewan with the second smallest percentage change overall (1.9 percent). Manitoba's increasingly positive trajectory resulted in a statistically significant increase over the 2013 to 2016 period for a modest overall percentage gain of 2.4. British Columbia performed comparatively weakly with the sixth lowest 2016 provincial score of 494, significantly lower than Ontario, Alberta, and Canada. Although B.C. scores traced a steadily positive trajectory over time, increases amounted to a modest 2.7

percentage gain over the three assessments. Alberta's scores followed a similar pattern for a more modest increase of 2 percent, but Alberta's higher 2016 score built on a 2010 baseline score 14 points higher than B.C. Nova Scotia achieved the largest sustained improvement after Quebec with significant assessment-over-assessment gains for a 4.9 percent increase overall. Scores in New Brunswick followed an increasingly positive trajectory with 18 of the total 20 score point gain being achieved over the final two assessments.

Results from the last three PISA assessments in 2012, 2015, and 2018 are those most directly comparable to the available PCAP results. Indeed, the Grade 8 student cohorts sampled by PCAP form the bulk of the PISA sample two years later. Figure 6 compares percentage changes in PCAP and PISA scores over these three comparable periods, with the provinces ordered by increasing percentage changes in PCAP scores. The discrepancies between the PISA decreases and PCAP increases are evident but, with some exceptions, the magnitude of these discrepancies is reduced by the smaller percentage change in the three most recent PISA assessments than those over the six assessments charted in figure 3. Prince Edward Island is one exception. In this case, the changes in both PCAP and PISA are positive, with a large difference between the two positive score estimates. Given the very small student population on P.E.I., this discrepancy, along with the volatility of P.E.I.'s PISA scores over the longer time span as noted earlier, may be influenced by sampling issues. In figure 6, only Saskatchewan, British Columbia, and New Brunswick have declines greater than two percentage points in the three most recent PISA assessments, the remaining provinces having percentage declines of two points or less. The essentially negligible difference in the Ontario's PCAP and PISA results is especially notable given the schools in this province enrol well over a third (38.9 percent) of all Canadian students.

Figure 6: Percentage change in PCAP 2010-16 Grade 8 and PISA 2012-18 Grade 10 scores, Canada and provinces



Sources: See tables 2 and 4.

Discrepant results

The best explanation for the discrepant provincial score trajectories in the PCAP and PISA results is that the two assessment instruments are measuring different things. Most obviously, PISA is designed to assess the performance of 15 year old, lower secondary school students, most of whom are enrolled in Grade 10, while PCAP assesses Grade 8 students enrolled in either junior high or, in the case of most Ontario students, the senior year of elementary school. In addition to the important structural and cultural differences in the teaching-learning environments in secondary and middle/senior elementary schools, curriculum expectations vary. These differences, together with the differing international and national orientations of the PISA and PCAP assessments respectively, mean that the two instruments have been designed within different theoretical frameworks to assess different aspects and applications of mathematical proficiency.

This view is supported by accounts in the technical literature for the two projects (OECD, 2019b; O'Grady and Fung, 2016). The PISA mathematical framework focuses on assessing "mathematical literacy" defined as "an individual's capacity to formulate, employ and interpret mathematics in a variety of contexts" (OECD, 2019b: 75). Assessment items focus on applying mathematical concepts and capabilities to understand and respond to meaningful real-world problems. While PISA recognizes that this approach is "closely aligned with that typically found in national mathematics curricula," (OECD, 2019b: 83), it is not designed to directly measure student proficiency in the subjects and strands contained in official curriculum as is the PCAP project, but rather to demonstrate ability to function mathematically in the real world. As declared by O'Grady et al. (2018: 8), PCAP is intended to serve as an "Assessment of learning" by providing "a snapshot of student achievement relative to specific curriculum requirements." More specifically, "The PCAP Mathematics Assessment focuses on curricular outcomes that are common to all Canadian provinces at the Grade 8/Secondary II level" (O'Grady et al., 2018: 35). In sum, while the PISA test items are necessarily framed within and oriented to a generalized international context, PCAP items draw on curriculum content common to the Canadian provinces. Moreover, and importantly, PCAP is geared to expectations for Canadian Grade 8 students while PISA seeks to assess the proficiency of lower secondary school students who have had a further two years or so of instruction.

In addition to offering a plausible explanation for the discrepancies between the PCAP and PISA results, this view suggests that increases in PCAP scores over the 2010 to 2016 period can be reasonably attributed to improved harmonization of test items, provincial curricula, and instruction within the Canadian context. The absence of a similar direct linkages between item design, official curricula, and classroom instruction in the design of the PISA assessment removes these potential influences on measurement outcomes.

The differing purposes and foci in the PISA and PCAP assessments result in a lack of commensurability. Rather than measuring a common underlying standard of mathematical proficiency among children at the same stage of development and schooling, each measures different aspects of mathematical meaning and ability within different performance contexts. There are nonetheless underlying commonalities, so that while

the respective measurements are not fully commensurable, they are complementary, as reflected in the similar relative ranks of the provinces. In one sense, poor commensurability impedes our quest for consensus on the direction and degree of changes in Canadian math scores, yet the two measures broaden the tentative conclusions available by yielding results at two different points on the ladder of formal schooling, PISA assessing Grade 10 performance, PCAP Grade 8.

Summary

Figure 7 plots the most recent provincial results from the three measures considered in this section against each other. Scores on all three measures are scaled to their standardized means of 500 with standard deviations of 100. Confidence intervals for PISA and TIMSS scores are omitted to avoid clutter.

As summarised in figure 7, the relative levels of provincial performance largely hold across the most recent PISA, PCAP, and TIMMS results. Indeed, there is a high correlation between aggregate 2016 PCAP and 2018 PISA scores (r.(8) =.889, p. < .01) and the linear regression line shows a good fit, accounting for almost 80 percent of the variation with a significant β (1) value (8.826, t(9)=4.459, p.<.001). Quebec stands out in all four sets of results as the preeminent performer, clearly outscoring all provinces on all measures. Ontario and Alberta share a statistical tie for second place in the 2016

Figure 7: Scatter plot of PISA 2018, PCAP 2016, and TIMSS 2015 estimated average provincial math scores

Note: Dotted line is best fit linear regression trend line for 2018 PISA scores against PCAP scores (B(1)=8.826, t(9)=4.459, p.<.001). Solid blue line shows statistical smooth line through 2018 PISA scores.

Source: See tables 2, 3, and 4.

PCAP results but, as shown earlier in figure 2, the larger confidence intervals in the 2018 PISA data include British Columbia and Nova Scotia in the statistical tie for second place in those results. There is a sharper differentiation in the 2016 PCAP data, with British Columbia's estimated score being statistically lower that Alberta and Ontario, but similar to those of the Atlantic provinces. Manitoba and Saskatchewan fall at the lower end of both distributions with statistically indistinguishable scores. The statistical smooth line connecting PCAP scores highlights the graphically anomalous results for Prince Edward Island with a difference of 16 standardized score points between the PCAP 2016 and PISA 2018 results.

Overall, the pattern of score distributions in figure 7 divides the provinces into three groups. Quebec is the sole occupant of the premier performing group. Ontario and Alberta comprise the second highest performing group as defined by shared PISA and PCAP results, and reinforced in the case of Ontario by the 2015 TIMSS results. In general, the remaining provinces fall into a statistically undifferentiated lower performing group. Manitoba and, to a less precise degree Saskatchewan, could be separated out as a fourth group on the basis of PCAP scores, but are to be included with the Atlantic provinces on the basis of 2018 PISA results given the wider confidence intervals in the PISA data. Similar considerations apply to Newfoundland and Labrador which could be distinguished from the other Atlantic provinces (as well as Saskatchewan) in the PCAP results, but not in the PISA data. On this analysis, Prince Edward Island's graphical outlier status is negated by overlapping PCAP confidence intervals with the other Maritime provinces (and B.C.), as well as overlapping PISA confidence intervals with all third group members. Together, the seven provinces in the lowest performing group enrol almost a quarter (24.3 percent) of Canadian school students.

As discussed earlier, the most notable discrepancies between the measures considered in this section concern changes over time. Whereas the PISA results show decline, PCAP results show improvement. If, as seems reasonable, this is a result of each test measuring different forms of mathematical performance at different grade levels, informed by different theoretical frames for different purposes, then each set of results needs to be interpreted with reference to these different purposes. Given that the PCAP assessment is geared to provincial curricula while PISA seeks a more global assessment, it seems reasonable to conclude that Canada is gaining ground locally while falling behind internationally. More specifically, the average mathematical proficiency of Canadian students has improved when measured against provincial curriculum expectations, but has declined when measured against the most widely accepted international standards. Despite this decline, Canada has continued to outperform other comparable countries. Still, Canada's impressive international record rests heavily on Quebec's particular prominence. This implies that the increasing PCAP math scores in other provinces may foster a fragile complacency which distracts from noticing and addressing possible weaknesses in curriculum design and delivery that may be slowly eroding our PISA results.

Provincial Assessments

Provincial measurement systems

Each province has a unique student assessment system, precluding cross-provincial comparisons. When considering the range and variety of systems in place an initial distinction can usefully be made between progress assessments conducted at designated points in students' progression through the curriculum, and cumulative examinations held toward or at the end of a school program, the results from which typically contribute a defined portion to final course marks. All provinces except Quebec and Saskatchewan report provincial progress assessments for mathematics, but these are conducted at differing grade levels. Alberta measures math performance in Grades 6 and 9, British Columbia in Grades 4 and 7, while Nova Scotia is currently implementing new math assessments in Grades 4, 6, and 8. These provinces also administer culminating examinations in math, with British Columbia recently replacing its Grade 10 math exam with an on-line Grade 10 Graduation Numeracy Assessment. In contrast, Ontario administers progress assessments in math in Grades 3, 6, and 9, but does not have cumulative provincial examinations for math, although students must meet a Grade 10 literacy requirement to graduate.

Quebec has a unique hybrid system of compulsory examinations in Grades 4, 6, and 8 (Secondary II). Exams are set centrally but administered in each school under uniform conditions, then marked locally by the teachers. Results are used to guide instruction but also count for 20 percent of students' final marks. Consequently, there are no provincial level assessment data for students in the lower and middle grades in Quebec. Alberta is phasing in a similar Grade 3 Student Learning Assessment program in literacy and numeracy where the student response tasks are also managed centrally but marked locally, with results intended for teacher and school use. Alberta's new SLA program is distinct from the progression assessment results provided by its Provincial Achievement Tests in Grades 6 and 9.

Provincial progress and cumulative assessments seek to assess each eligible student to achieve complete coverage of designated student populations and provide individual results for students, teachers, schools, and parents, as well as aggregated board and provincial results. Various exceptions are permitted, but the aggregated results are accepted as census measures of the populations concerned, rather than estimates derived from population samples, as in the international and national assessments reviewed earlier. Culminating examinations normally focus on the content of specific secondary courses and will thus cover only students enrolled in each course.

All progress assessments use criterion referenced measures expressed as the percentage of students at a predetermined standard of achievement. The declared provincial standard of achievement sets the milepost which students are expected to have reached or exceeded by that year. Those who meet this standard are considered to be on pace with curriculum expectations. The sum of those who meet and exceed the expected standard

relative to the sum of all students is taken to approximate mathematical performance. Even if achievement standards in different provinces could be accepted as commensurable, the categorical classification of student performance into the categories of "does not meet the standard," "meets the standard" and/or "exceeds the expected standard" is not well suited for comparison, as it does not tell us the true cardinal performance distances.

The broad picture

A detailed summary of 36 different provincial measures of math performance is included in the Appendix, together with selected trend indicators. The variety of assessments and the results provide a bewildering picture. Missing data also limits comparative analysis, especially the lack of progress measures in Quebec and Saskatchewan, and the lack of cumulative secondary level measures in Ontario. Even so, trends within the provincial results cohere in several ways with trends in the PCAP and PISA results reviewed earlier.

Table 5 summarizes the trends reported in the Appendix under three curriculum levels, Elementary (Grades 3 to 6), Intermediate (Grades 7 to 9), and Secondary (Grades 10 to 12). Trend trajectories shown in the Appendix are collapsed into single summative trend indicators. Thus, the U shaped and positive flattening trajectories for Alberta's two Grade 12 exams in the Appendix are condensed to Positive in table 5, and so forth. [13] Provinces are ranked by decreasing 2016 Grade 8 PCAP scores, with 2018 PISA ranks shown in the final column.

Table 5: Summary of trends in provincial math assessments over varying periods

Provinces ranked by 2016		PISA 2018		
PCAP scores (Table 4)	Elementary (3-6)	Intermediate (7-9)	Secondary (10-12)	rank (Table 3)
1. Quebec	No data	No data	Positive	1
2. Ontario	Negative	Positive	No data	2
3. Alberta	Flat	Negative	Positive	3
4. Prince Edward Island	Flat	Flat	Positive	8
5. New Brunswick	Insufficie	nt data to estima	te trends	6
6. Nova Scotia	Flat	Flat	Flat	5
7. British Columbia	Flat	Negative	Flat	4
8. Newfoundland & Labrador	Flat	Negative	Flat	7
9. Saskatchewan	No data	No data	Flat	9
10. Manitoba	Positive	Positive	Flat	10

Note: Bold typeface indicates at least one significant $\beta(1)$ linear regression coefficient (p.< .05) for the long term score changes reported in Appendix.

^[13] The Grade 3 and Grade 6 trend indicators for both Prince Edward Island and Newfoundland and Labrador were contradictory. In each case these were condensed to Flat after inspecting line graphs of merged data.

Each of the four highest PCAP scoring provinces and each of the three highest scoring PISA provinces has a Positive internal score trajectory in its provincial assessments at either the Intermediate or Secondary curriculum levels. Alberta has a conflicting Negative trend at the Intermediate level. Ontario has a Negative score trend in its Elementary assessments with both of the province's Grade 3 and 6 scores having statistically significant but shallow negative scores trends. These coincide with the province's flat Grade 4 TIMSS profile.

Where results were available, lower scoring PCAP and PISA provinces have either Flat or Negative provincial score trends in table 5, with the sole exception of lowest ranked Manitoba which has statistically significant Positive trends at both Elementary and Intermediate levels. The significant Intermediate level trend in the internal data is inconsistent with both the province's PCAP and PISA profiles. British Columbia's statistically significant negative trend at the Intermediate level is also inconsistent with the significant increase in PCAP scores reported in table 4. Prince Edward Island's solid performance in table 5 complements its PCAP ranking, but appears inconsistent with its PISA record, underscoring the irregular pattern shown in figure 7. Consequently, P.E.I. has a positive record on all measures except PISA, but even there the province's gains on the up side of the U-shaped recovery in recent results makes a positive contribution to its commendable overall record.

Provincial assessment results in selected provinces

We have selected results from Quebec, Ontario, Alberta, and British Columbia for closer consideration because of their prominence in the PISA and PCAP results and their relatively large enrolments. Provincial assessments in each of these provinces also illustrate one or more aspects of interest shared with other provinces.

Quebec

The Ministère de l'Éducation, de l'Enseignement supérieur et de la Recherche (MEESR) administers three compulsory Secondary IV (Grade 11) Ministerial examinations in mathematics as part of Quebec's Secondary School Diploma requirements: (1) the Cultural, Social and Technical Option (3414) taken by an average of 49 percent of students sitting these exams over the seven years summarized in the Appendix, (2) the Science Option (5426), accounting for an average of 41 percent of students, and (3) the Technical and Scientific Option (4426) taken by an average of 10 percent of students over this period. Exam materials are distributed by MEESR and completed in schools on a set day under controlled conditions. Grading is divided between schools and the MEESR (Quebec, 2015).

Separate examinations are set for schools in Quebec's French and English sections. On average, 88 percent of the three math exams over the 2012/18 period were French language exams. The marks reported in the Appendix are weighted averages for the results in each language. Figure 8a plots these weighted average scores together with best fit linear regression lines for the Beta values in the Appendix. The positive mark trajectories for all three sets of examination results are evident, as is the drop in scores for the 5426 Science Option from 2016 to 2017, followed by a partial recovery which formed the basis for assigning a positive but flattening trajectory in this case.

Figure 8a: Quebec Ministerial math exam results, French and English weighted average

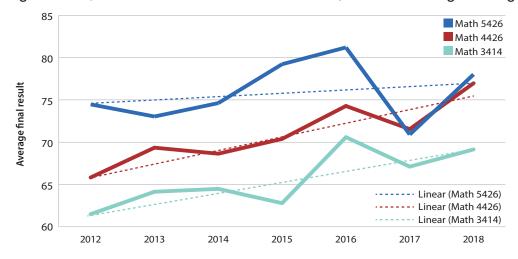


Figure 8b: French - English score difference on Quebec Ministerial math exams

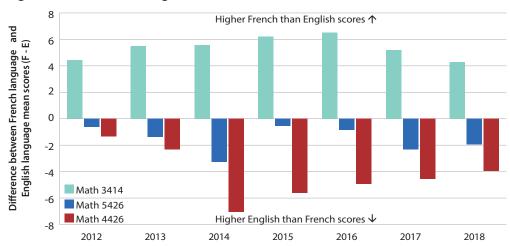
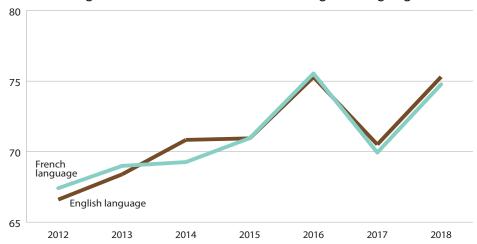


Figure 8c: Average marks on three French and English language math exams



Source: Data provided by Direction des indicateurs et des statistiques on request (30 June, 2020) and authors' calculations.

Scores on the two language versions of each exam were highly correlated, [14] but there were higher average scores on the French language version of the 3414 CST exam and higher average scores on the English language version of the 4426 TS exam. French and English annual average score differences for each exam are compared in figure 8b, values above the zero line showing higher French than English scores, those below the zero line higher English language scores. Nevertheless, score differences between French and English language exams are effectively cancelled out when scores for all three courses are averaged, as shown in figure 8c. Linear regression of the grand average of scores on all three courses in both languages showed a statistically significant positive trend across the years considered. [15]

In sum, results from Quebec's three culminating math exams from 2012 to 2018 are consistent with the stable or improving scores from international and national assessments. The language-based differences are only partially consistent with other findings. French language Quebec students in both the Grade 8 PCAP and 15 year old PISA math assessments have consistently outperformed their English language peers (CMEC, 2018: 37). The internal exam data supports this pattern for students sitting only the most highly subscribed of Quebec's three culminating math examinations.

Ontario

Ontario's Education Quality and Accountability Office (EQAO), an arm's length Crown corporation operating independently of the Ministry of Education, administers progress examinations in mathematics in Grades 3, 6, and 9 to all public school students. As in Quebec, students in the English and French language systems sit separate forms of each exam with results reported separately, but there are notably fewer minority language students, accounting for an average of just 4.5 percent of the Ontario results summarized in the Appendix, compared to the 11.7 percent in Quebec.

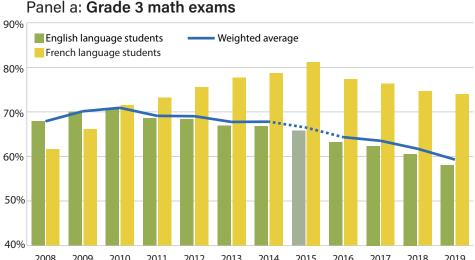
The Grade 3 and 6 results summarized in the Appendix are weighted averages of the English and French language results. Score trajectories for both grades show statistically significant declines, Grade 3 results experiencing a 13 percent decease over the 2008 to 2019 period, and Grade 6 results a more substantial 19 percent decline. Figures 9a and 9b chart results for English and French language students in each grade respectively with a line plot tracing the weighted average. With English language students accounting for slightly more than 95 percent of total results, the increasingly negative score trajectories for English language students in Grade 3 and the flattening negative trajectory for Grade 6 students both match the overall score trajectories in the Appendix. Yet the graphs show that these overall negative trends mask positive gains for French language students in both grades. Grade 3 French language scores increased 32 percent over the 2008 to 2015 period to peak at 81 percent of students at or above the provincial standard, before declining

^[14] Ranging from r(5)=.968, p.<.001 on the 3414 Culture, Social and Technical (CST) option, to r(5)=.910, p.<.01 on the least popular 4426 Technical and Scientific (TS) option.

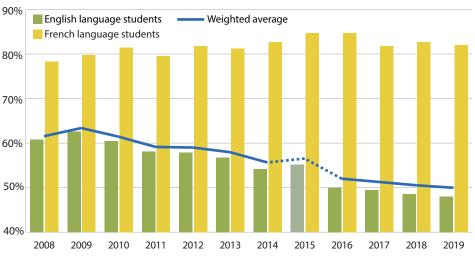
^[15] $\beta(1) = 1.161, t(6) = 3.007, p. = .03.$

to 74 percent at this standard in 2019. Most strikingly, Grade 3 English language students outperformed French language students in 2008 and 2009 before falling increasingly behind in subsequent years (figure 9a). In Grade 6 (figure 9b), French language students consistently and substantially outperformed their English language counterparts, with 78 percent of students at or above the provincial standard in 2008, rising to 82 percent in 2019. In contrast, the percentage of English language students at or above the provincial standard declined steadily from 61 percent in 2008 to 48 percent in 2019.

Figure 9: Percentage of Ontario English and French language students at or above provincial standard



2008 2009 2010 2011 2012 2013 2019 2014 2015 2016 2017 2018 Panel b: Grade 6 math exams



Notes: Years are the latest in the academic pair, so that 2018 corresponds to 2017/18. Missing English language results due

to exceptional circumstances in 2015 replaced with imputed values (multivatiate normal method). Source: Ontario Education Quality and Accountability Office (EQAO) and authors' calculations.

Ontario Grade 9 students enrol in either the Academic or Applied mathematics course and consequently participate in one of two separate assessments. [16] The summary results in the Appendix show weighted averages for combined English and French language students participating in each assessment. In contrast to the lower grades, results in both the Academic and Applied Grade 9 assessments show statistically significant positive but flattening score trajectories, as shown in figure 10a. This graph also shows a substantial score gap between the two assessments. Many more students take the Academic assessments than the Applied assessments, and the ratio has been increasing so that while 68 percent of Grade 9 students took the Academic level test in 2008, this proportion had risen to 75 percent in 2019. To provide a balanced overview of Grade 9 results, figure 10a also includes weighted average results for Academic and Applied students. This weighted average shows a statistically significant 20 percent increase over the 2008 to 2019 period.

There are two interesting things to point out. The first is that the course-based differences in the Grade 9 results could simply be the product of less proficient students sorting themselves into the less demanding course. This would create a composition bias whereby mildly above average students choosing to enrol in the less demanding course would boost the average performance in that course while simultaneously improving test results in the more demanding course by concentrating contributions from the top students. In this case, average marks in both groups go up even if the overall distribution of student proficiency has not changed. This effect will disappear with the recently announced destreaming of the Grade 9 program. The resulting effect on Grade 9 math performance will also be influenced by accompanying curriculum changes.

Figure 10b breaks out results for English and French language students in Academic and Applied courses. Score trajectories for Grade 9 English and French language students are positively correlated [17] and fall within similar ranges within each course. In accord with the overall results, trends for all four score trajectories are positive, but with slightly steeper increases for French language students in each course. In the Academic assessments, English language results are essentially flat after 2010 while French language results continue to increase to a peak of 90 percent of students at or above the provincial standard in 2019, compared to 84 percent for English language students. Scores for Applied students follow a similar, but less pronounced, pattern, with French language students outperforming English language students in recent years, with the sole exception of 2017 when scores were tied.

^[16] The Minister of Education announced the impending end of this policy in a July 9, 2020 statement to the legislature (Ontario, 2020). A new foundational Grade 9 math course will be introduced for September 2021. While this change will necessarily be integrated into Ontario's new math strategy, the announcement linked the new initiative to the government's commitment to improving education equity.

^[17] Academic English and French language r.(10)=.748, p.=.005; Applied English and French language r.(10) = .817, p.=.001.

Grade 9 Academic Grade 9 weighted average

Grade 9 Applied

80%

60%

40%

20%

2013

2009

2008

2010

2011

2012

Figure 10a: Percentage of Ontario Grade 9 Academic and Applied students at or above the provincial standard

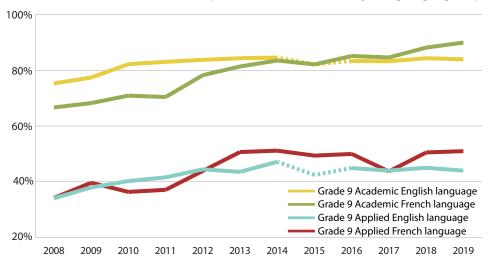


2014

2015

2016

2017



Notes: Years are the latest in the academic pair, so that 2018 corresponds to 2017/18. Missing English language results due to exceptional circumstances in 2015 replaced with imputed values (multivatiate normal method).

Source: Ontario Education Quality and Accountability Office (EQAO) and authors' calculations.

Ontario's internal Grade 9 results do not conform to the international and national results discussed earlier. In the PISA data, Ontario experienced an overall 2.5 percent decline in average mathematics test scores among 15 year olds over the 2009 to 2018 period, while the weighted average of students achieving or exceeding the provincial standard in Grade 9 Academic and Applied courses over the same period increased by 15 percent. We ought to be careful not to infer too much from this discrepancy. Ontario's own assessments are based on the distribution of students' marks across and within the

categories of not meeting and at or above the provincial standard. If the increase in the proportion of students meeting or exceeding the standard in a test year comes from the inclusion of students who would probably have fallen slightly below the standard in previous years, any increase in measured scores would be relatively small. To complicate matters further, all students selected through PISA's sampling procedures will normally sit the test, regardless of their enrolment in Academic or Applied courses. Moreover, Ontario's policy of charging fees for student assessment participants in independent (non-public) schools excludes an unknown proportion of Grade 9 students from the EQAO results. Finally, as discussed more generally when comparing PISA and PCAP outcomes, the goals and philosophy of Ontario's secondary level mathematics curriculum are likely not fully compatible with those of PISA, so that gains on the provincial assessments will not necessarily be accompanied by similar gains in the PISA data. In this respect, Ontario's impeding abandonment of Grade 9 streaming will include a curriculum review which will provide opportunities to better harmonize the program of studies to the PISA framework, if considered desirable.

In sum, the outcomes of the criterion-referenced EQAO and item-scored PISA tests are not commensurable and we must be prepared to make appropriate allowances when comparing results. In this case, it would not be unreasonable to interpret the PISA Grade 10 and EQAO Grade 9 data as reflecting largely stable to somewhat positive math performance by Ontario lower secondary level students over comparable time periods. This interpretation conforms to the Grade 8 PCAP results, as does the superior performance of French language students.

The elementary level EQAO results largely match the Grade 4 TIMSS data, with some reservations. The TIMSS results span the 2003 to 2015 period in four-year steps, while EQAO data provides annual coverage from 2008 to 2019. The first three Grade 4 data points in the TIMSS results show essentially flat performance with a slight nonstatistically significant variation from 2003 to 2011, followed by a small but again nonsignificant drop from 2011 to 2015. This compares to essentially stable English language Grade 3 EQAO results from 2008 to 2011, followed by small, slightly accelerating, declines thereafter resulting in a statistically significant decline over the full 2008 to 2019 period. The trajectory of the EQAO English language Grade 6 results is similar, but more pronounced. Bearing in mind the difficulties in comparing criterion-referenced proportions with standardized scores, both data sets can be interpreted as showing largely steady performance over comparable years, with declines becoming more evident in the more recent EQAO data. The soon to be published TIMMS 2019 results loom large at this point. If they reveal a continuing decline in Grade 4 scores to match the decreasing proportions of Grade 3 and 6 students achieving the EQAO standard, this will not only support this trend, the TIMMS data may also provide independent support for a statistically significant decline in Ontario's elementary math scores since 2011. As noted at the outset, Ontario has already committed to a revised program of studies in math as a result of concern over the declining EQAO results (Ontario, 2019).

Alberta

Alberta Education administers Provincial Achievement Tests (PATs) in mathematics in Grades 6 and 9. As summarized in the Appendix, results show mild declines in achievement over the 2011 to 2019 period. As illustrated in figure 11a, the score trajectory for Grade 6 students at the acceptable standard was flat over 2011 to 2019 period with a minor decline and temporary drop in 2016 and 2017 respectively. Grade 9 scores suffered a more marked drop in 2018 and 2019, tracing an increasingly negative trajectory overall. As summarized in the Appendix, while 76 percent of Grade 9 students met the provincial standard in 2016 only 67 percent did so in 2019, for a percentage decline of 10.7.

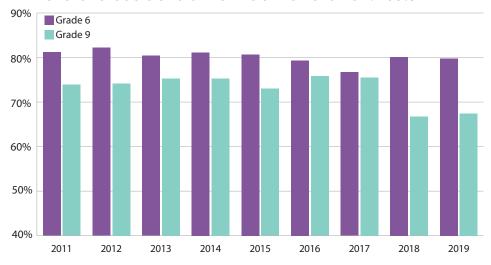
Results from Alberta's Grade 12 diploma exams are more volatile. The M30-1 exam focuses on course content intended to prepare students for university level studies in mathematics-intensive programs requiring calculus, while course content for the M30-2 exam is less demanding. On average across the years summarized in the Appendix, the M30-1 exam accounted for a 61 percent of Alberta's Grade 12 math exams. As shown in figure 11b, the percentage of students achieving the acceptable standard on this exam followed a shallow U-shaped trajectory over the period considered, falling from a high of 81 percent in 2013 to a low of 71 percent in 2016, before recovering to 78 percent in 2018 and 2019. Results in the less demanding and less popular M30-2 course followed a positive but flattening trajectory.

Alberta reports results as the percentage of students achieving the acceptable standard and those achieving a higher "standard of excellence." The ratio of students at the two standards is revealing of overall performance. For example, the percentage of students who met the standard of excellence in Grade 6 fell from 20 percent in 2011 to 14 percent in 2017, before recovering to 17 percent in 2019, while overall performance as measured by students achieving the acceptable standard remained stable. This suggests that there was a compression at the top of the achievement range where the best students continued to outperform the rest but by a declining, and then partially recovering, margin. In the Grade 9 scores the reverse happens. There is a small overall increase of 2 percentage points in the share of students who met the standard of excellence, while the proportion of those achieving the expected standard fell by 7 percentage points. This suggests students at or above the excellent standard are hiding a non-negligible cohort of students falling just under the cut-off point.

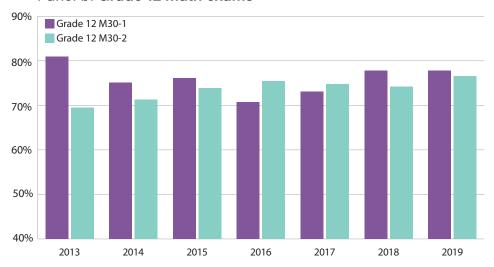
Alberta's overall scores are largely consistent with the PISA trends reviewed earlier. While Alberta experienced a marked decline in PISA scores, most of this decline (19 out of 38 score points) occurred between 2003 and 2006, with a less precipitous drop of 13 score points from 2006 to 2012. This more recent, largely stable, pattern reflects what is observed in the provincial assessments, with the exception of the drop in Grade 9 PAT results in 2017 and 2018.

Figure 11: Percentage of Alberta students at the acceptable standard

Panel a: Grade 6 and 9 Provincial Achievement Tests



Panel b: Grade 12 math exams



Note: Years are the latest in the academic pair, so that 2018 corresponds to 2017/18.

Source: Alberta, 2019.

British Columbia

The Foundation Skills Assessment (FSA) administered by the Ministry of Education provides an annual assessment of numeracy skills in Grades 4 and 7. Recent changes have made results from 2018 onward less than fully comparable with those from earlier years, limiting the summary results considered in the Appendix to the 2008 to 2017 period, although subsequent years are included in the B.C. graphs below for illustrative purposes. As summarized in the Appendix and shown in figure 12a, Grade 4 results hovered around 77 percent of students meeting or exceeding expectations, touching highs of 80 percent in 2012 and again 2016 before dropping to 77 percent in 2017, for a flat score trajectory overall. Grade 7 results experienced a two step decline from 77 percent at or above the provincial standard in 2008, to 73 percent in 2012, followed by a partial rebound before further decline to a low of 72 percent in 2017, yielding a statistically significant negative, but flattening, score trajectory. The Appendix also summarizes results for British Columbia's now discontinued Grade 10 Foundations of Mathematics and Pre-Calculus provincial examination, [18] where scores followed a flat trajectory with minor variation around the mean.

British Columbia also reports FSA results as average scores, as plotted in figure 12b, which illustrates important differences from the categorical data in figure 12a. Once a student has been assigned to a category by falling above or below a predetermined achievement criterion, we are prevented from telling how far from that benchmark he or she is. When applied to large numbers of students, we cannot tell how students are distributed below or above the set criteria. Thus, it could be that a high proportion of students are clustered just below the cut-off point, are spread out more widely below that point, or follow some other distribution. Plotting average scores as in figure 12b helps address this limitation by showing the numbers of students at more discrete, equally scaled achievement levels. As a result, the initial decline, recovery, and then shallower decline of Grade 7 2008 to 2017 results noted above are cast into sharper relief in figure 12b, as are the annual variations in Grade 4 scores. Including the adjusted average scores for 2018 and 2019 in figure 12b also modifies the picture to suggest more of a continual decline for Grade 4 scores and a partial recovery for Grade 7.

British Columbia reports FSA results for public and independent (private) schools. As shown in figures 13a and 13b there are substantial differences between average student marks in the two types of school at both grades. On average over the 2008 to 2019 period, students in independent schools outperformed students in public schools by 69 score points in Grade 4 and 68 score points in Grade 7. Over this period, the Grade 4 public school average fell by 9 points, but increased by 10 points in independent schools. There was a similar pattern in the Grade 7 results as shown in figure 13b, the average Grade 7 score in public schools falling by 11 score points, but increasing by a more modest 3 score points in independent schools.

^[18] The most popular of the four Grade 10 math exams over the 2011/16 period, accounting for 82 percent of reported marks.

Figure 12a: Percentage of Grade 4 and 7 British Columbia students meeting or exceeding expectations in numeracy

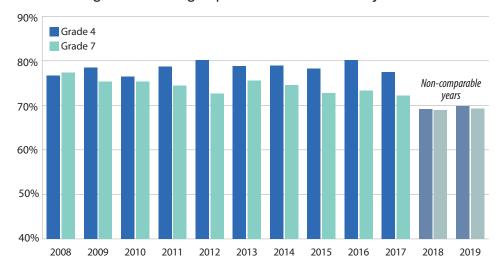
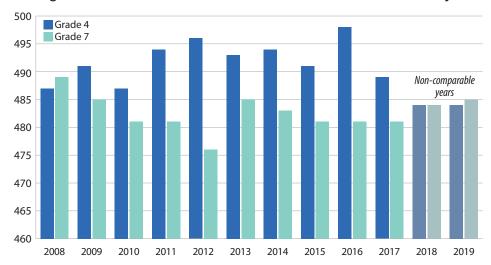


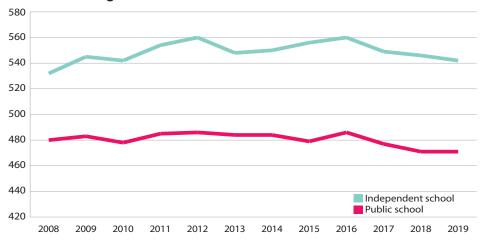
Figure 12b: Mean Grade 4 and 7 British Columbia numeracy scores



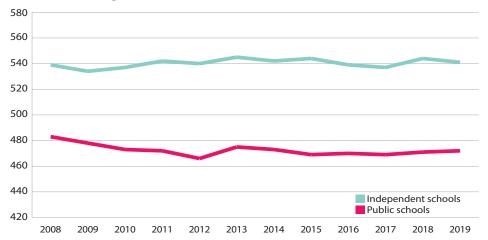
Note: 2018 and 2019 results not directly comparable with earlier years due to change in proficiency levels. Source: British Columbia, 2019,

Figure 13: Numeracy scores in British Columbia, independent and public schools

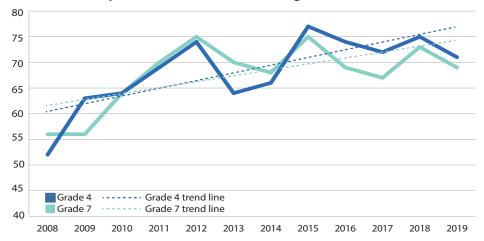
Panel a: Average scores, Grade 4



Panel b: Average scores, Grade 7



Panel c: Score point differences in average Grade 4 and 7 numeracy results



Note: 2018 and 2019 results not directly comparable with earlier years due to change in proficiency levels. Source: British Columbia, 2019,

Annual differences between the average scores in both kinds of schools are graphed in figure 13c, showing a steady increase in the achievement gap between students in these schools in both grades. This implies public schools are falling behind independent schools in effectively teaching math as measured on these tests. This further suggests the overall Grade 7 decline reported in the Appendix and shown in figure 12b is primarily attributable to public rather than independent schools. These are important findings because they show the relatively higher achievements levels in independent schools moderating downward trends in math scores and boosting overall provincial performance.

Similar patterns may be present in other provinces. Summary results for Quebec's Ministerial Secondary V examinations in all subjects [19] over the 2007 to 2011 period, for instance, showed an average seven percent point advantage for independent schools (Quebec, 2012: Table 2). No comparable data are available for Ontario where, with the exception of the Grade 10 literacy test, independent schools are not required to participate in the EQAO assessments and the policy of charging independent schools a perstudent participation fee deters many from doing so.

^[19] Mathematics was not included in this set of subjects, which focus on Language at this level.

Conclusion

On balance, there are good reasons to conclude that mathematical proficiency among Canadian students is in decline. Care has to be taken in drawing inferences from the available assessments of mathematical achievement, yet a largely consistent picture emerges from the preceding analysis. Results from multiple sources show Quebec Grade 8 and Grade 10 students consistently outperforming those in all other provinces over the past decade and more. Ontario, Alberta, and British Columbia collectively occupy a broadly defined second place in the Grade 10 Programme for International Student Assessment (PISA) results, with British Columbia occupying a lower rank in results from the Grade 8 Pan-Canadian Assessment Program (PCAP). Quebec's continuing prominence rests on stable or increasing scores in comparison to declines elsewhere. Alberta and British Columbia placed above or around the national average in earlier PISA results, but scores in both provinces subsequently declined markedly. Ontario Grade 10 PISA scores declined less steeply than those in Alberta or British Columbia, while Ontario's Grade 8 PCAP scores remained stable. Scores in the Atlantic and remaining prairie provinces have remained below the Canadian average on both measures, with a more volatile and promising performance record from Prince Edward Island. With the exception of contrary PCAP results discussed below, the overall pattern is one of a disturbing decline in secondary and middle school measures of math achievement. Canada is not alone in this respect, the most recent PISA data revealing widespread declines in secondary level math scores across OECD countries.

Declining scores have lowered Canada's international standing from fifth place overall in the 2003 PISA rankings, to twelfth place in the most recent 2018 results. This decline has been accentuated by the rise of Asian jurisdictions in recent assessments. Even so, Canada still scores well above the OECD average and ranks second behind Japan among G7 countries. Given the pattern of declining provincial scores this credible standing is largely attributable to Quebec's contribution to the national score.

Trends in math performance in elementary grades are less clear due to a lack of comparable data. Results from the sole admissible but limited international data source (TIMSS) conform to the secondary and middle grade rankings, with Grade 4 students in Quebec significantly outperforming those in Ontario, who outperformed those in Alberta.

Results from the many provincially designed and administered assessments and examinations cannot be compared with each other, but internal trends in these results paint a generally complementary picture. Secondary level exam results follow either positive trajectories in Quebec, Alberta, and P.E.I., or flat trajectories in the remaining five provinces for which data were available. Middle grade test results have positive trajectories in Ontario and Manitoba, negative trajectories in Alberta, British Columbia and Newfoundland and Labrador, or remain flat in P.E.I. and Nova Scotia. Elementary

score trajectories are also flat in five of the seven provinces with useable data, negative in Ontario, and positive in Manitoba.

A discordant note is struck by trends in provincial results from the Grade 8 Pan-Canadian Assessment Program, Canada's only home-designed national assessment. Math scores in all provinces except Ontario increased significantly over the three PCAP cycles from 2010 to 2016, while PISA scores declined. This appears attributable to differences between the design and purposes of PCAP and PISA, with the Canadian assessment aimed at measuring the performance of middle school students against common provincial curriculum expectations, and PISA seeking to assess the mathematical literacy of lower secondary school students within an international framework. Relative provincial standings are nevertheless similar on both measures, with Quebec consistently and significantly outperforming all other provinces, followed by Ontario and Alberta.

The contrary trends in the PCAP and international measures are disturbing, prompting the question of whether the current design and purpose of PCAP should be reconsidered. The improving performance of Grade 8 students in the current PCAP framework distracts from contrary trends in both international and some provincial assessments. Given that all provinces have their own assessment programs for mathematics (and other subjects), policy makers, teachers and parents could be better served by a more independent national assessment program informed by the broader standards used in international assessments. Key issues to be considered are the conceptual frame within which student performance is assessed, and the grade levels at which the assessments are conducted. A framework that complements, augments and extends provincial curriculum expectations by incorporating elements of the mathematical literacy approach adopted by PISA would have much to commend it. Continued assessment of Grade 8 students would also appear sensible, with the possible addition of Grade 4 students to provide a desirable estimate of developing proficiency at a crucial earlier age. Ideally, such a revised and extended assessment program would be conducted annually, or at least biennially.

Investing in a domestically developed and independently administered program of large scale mathematics assessment, solidly grounded in internationally anchored standards rather than those in provincial curriculum documents, would benefit teachers and parents, while providing policy makers with more widely based progress measures, especially as the provinces already have their own assessment programs geared to their curricula. More widely based assessment programs in other subjects would also be desirable, but the need for strong, reliable, and robustly comparative measures of mathematical proficiency among school students is preeminent given the importance of mathematical reasoning and application in modern economies.

References

Alberta, Ministry of Education (2019). *Provincial Achievement Test Multiyear Reports*. Government of Alberta. https://www.alberta.ca/provincial-achievement-tests.aspx#toc-2

British Columbia, Ministry of Education (2019). *BC Schools - Foundation Skills Assessment (FSA)*. Government of British Columbia. https://www2.gov.bc.ca/gov/content/education-training/k-12/administration/program-management/assessment/foundation-skills-assessment

Broer, Markus., Bai Yifan, and Fonseca Fonseca (2019). Methodology: Constructing a Socioeconomic Index for TIMSS Trend Analyses. In *Socioeconomic Inequality and Educational Outcomes. IEA Research for Education (A Series of In-depth Analyses Based on Data of the International Association for the Evaluation of Educational Achievement (IEA)*) volume 5. Springer. https://doi.org/10.1007/978-3-030-11991-1_3>

Council of Ministers of Education, Canada (2016). *PCAP 2016: Assessment framework*. CMEC. http://www.cmec.ca/537/Programs-and-Initiatives/Assessment/PanCanadian-Assessment-Program-(PCAP)/PCAP-2016/General-Information/index.html

Council of Ministers of Education, Canada (2014). *PCAP 2013: Report on the Pan-Canadian Assessment of Science, Reading, and Mathematics*. CMEC. https://www.cmec.ca/Publications/Lists/Publications/Attachments/337/PCAP-2013-Public-Report-EN.pdf

Council of Ministers of Education, Canada (2017). *TIMSS 2015: Canadian Results from the Trends in International Mathematics and Science Study*. CMEC. https://cmec.ca/Publications/Attachments/373/TIMSS2015_Report_EN.pdf

Council of Ministers of Education, Canada (2018). *Measuring up: Canadian Results of the OECD PISA 2018 Study The Performance of Canadian 15-Year-Olds in Reading, Mathematics, and Science.* CMEC. https://www.cmec.ca/Publications/Lists/Publications/Attachments/396/PISA2018_PublicReport_EN.pdf

Canadian Mathematical Society (2010, December 10). *Canada's Math Score a Cause for Concern*. Media Release. CMS. https://cms.math.ca/MediaReleases/2010/math-scores

Centrale des Syndicats du Québec (2011). Réaction CSQ aux nouvelles mesures en lecture : Une bonne nouvelle pour les enfants du Québec. CSQ.

Cordero, José M., Victor Cristobal, and Daniel Santín (2018). Causal Inference on Education Policies: A Survey of Empirical Studies Using PISA, TIMSS and PIRLS. *Journal of Economic Surveys* 32, 3: 878–915.

Fung, Karen, and Man-Wai Chu (2016). Fairness of Standardized Assessments: Discrepancy Between Provincial and Territorial Results. *Journal of Contemporary Issues in Education* 10, 1.

Goldstein, H. (2017). Measurement and Evaluation Issues with PISA. In Louis Volante (ed.), *The PISA Effect on Global Educational Governance* (Routledge): 49–58.

Grisay, Aletta, John H. A. L. De Jong, Eveline Gebhardt, Alla Berezner, and Beatrice Halleux-Monseur (2007). Translation Equivalence Across PISA countries. *Journal of Applied Measurement* 8, 3: 249–66.

Hammer, K., and C. Alphonso (2018, December 7). Tests Show Provincial Differences in Math, Reading, Science Education. *Globe and Mail*. https://www.theglobeandmail.com/news/national/education/tests-show-provincial-differences-in-math-reading-science-education/article20955151/>

Jerrim, John, Philip Parker, Alvaro Choi, Anna Katyn Chmielewski, Christine Sälzer, and Nikki Shure (2018). How Robust are Cross-Country Comparisons of PISA Scores to the Scaling Model Used? *Educational Measurement: Issues and Practice* 37, 4: 28–39. https://doi.org/10.1111/emip.12211

Klieme, Eckhard (2016). *TIMSS 2015 and PISA 2015. How are they Related on the Country Level?* German Institute for International Educational Research. https://pisa.dipf.de/de/pdf-ordner/Klieme_TIMSS2015andPISA2015.pdf

Logan, Nick (2014, January 9). Parents Across Canada Fight for Return to Traditional Math Lessons. *Global News*. https://globalnews.ca/news/1070386/parents-across-canada-fight-for-return-to-traditional-math-lessons/

O'Grady, Kathryn, and Karen Fung (2016). *PCAP 2016: Technical Report*. Council of Ministers of Education, Canada. https://cmec.ca/Publications/Lists/Publications/ Attachments/394/PCAP%202016%20Technical%20Report_FINAL_EN.pdf>

O'Grady, Kathryn, Karen Fung, Laura Serage, and Gulum Khan (2018). *PCAP 2016:*Report on the Pan-Canadian Assessment of Reading, Mathematics and Science. Council of Ministers of Education, Canada. https://cmec.ca/Publications/Lists/Publications/

Attachments/381/PCAP-2016-Public-Report-EN.pdf>

Organisation for Economic Co-operation and Development [OECD] (2019a). *PISA* 2018 Results (Volume I): What Students Know and Can Do. OECD. https://doi.org/10.1787/5f07c754-en

Organisation for Economic Co-operation and Development [OECD] (2019b). *PISA* 2018 Assessment and Analytical Framework. OECD. https://doi.org/10.1787/b25efab8-en

Ontario (2019, August 29). First Year Investment of Ontario's Four-Year Math Strategy Announced. Press Release. Government of Ontario. https://news.ontario.ca/en/release/53479/first-year-investment-of-ontarios-four-year-math-strategy-announced

Ontario (2020, July 9). *Ontario Taking Bold Action to Address Racism and Inequity in Schools*. Press Release. Government of Ontario. https://news.ontario.ca/en/release/57543/ontario-taking-bold-action-to-address-racism-and-inequity-in-schools-1

Quebec, Ministère de l'Éducation, de l'Enseignement supérieur et de la Recherche (2015). *Certification of Studies and Management of Ministerial Examinations:*Administrative guide-2015 edition. Government of Quebec. http://www.education.gouv.gc.ca/fileadmin/site_web/documents/dpse/sanction/Guide-sanction-2015_ang.pdf

Quebec, Direction de la sanction des études (2012). *Uniform Ministerial Examinations:* Results on the June 2011 Uniform Ministry Examinations. Government of Quebec. http://www.education.gouv.qc.ca/fileadmin/site_web/documents/dpse/sanction/Rapport-epreuves_2011_a_01.pdf

Roshowy, Kristin (2019, 28 August). Ontario Elementary Student Math Scores Continue to Decline, Latest EQAO Results Show. *The Star.* https://www.thestar.com/politics/provincial/2019/08/28/ontario-elementary-student-math-scores-continue-to-decline-latest-eqao-results-show.html

Statistics Canada (2020). *Table 37-10-0109-01: Number of students in elementary and secondary schools by school type and program type.* Statistics Canada. https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=3710010901>

Stokke, Anna (2015). What to Do about Canada's Declining Math Scores. C. D. Howe Institute Commentary No. 327. https://www.cdhowe.org/sites/default/files/attachments/research_papers/mixed/commentary_427.pdf

Volante, Louis (ed.) (2016). The Intersection of International Achievement Testing and Educational Policy: Global Perspectives on Large-Scale Reform. Routledge.

Zamarro, Gema, Collin Hitt, and Ildefonso Mendez. (2016). When Students Don't Care: Reexamining International Differences in Achievement and Student Effort. *Journal of Human Capital* 13, 4. https://scholarworks.uark.edu/edrepub/22>

Appendix

Overview of provincial mathematics assessments with selected trend indicators

The overview table below summarizes current math assessments in each province with selected results and trend estimates. Provinces are ordered alphabetically. The criterion column indicates the measurement standard used for the assessment data on the appropriate row. Progress assessments typically use categorically based measures expressed as the percentage of students at a predetermined standard of achievement. The declared standard of achievement sets the milepost which students are expected to have reached or exceeded by that year. Those who meet this standard are considered to be on pace with curriculum expectations. The sum of those who meet and exceed the expected standard relative to the sum of all students is taken to approximate mathematical performance. Even if achievement standards in different provinces could be accepted as comparable, the consequent categorical classification of student performance into the categories of 'does not meet the standard,' 'meets the standard,' and/or 'exceeds expected standard' is not well suited for comparison, as it does not tell us the true cardinal performance distances.

The table presents results for the most recent year available when compiled, for three years earlier, and for an earlier year determined by the data available. Where a (year) appears in a results cell that is the actual year for that specific result, overriding the year shown in the column heading. The three year and longer year spans shown in the table were used to calculate the Short Term (ST% Δ) and Longer Term (LT% Δ) percentage change statistics in the adjacent columns. The penultimate column reports the $\beta(1)$ regression coefficient (Beta value) for the slope of the best fit linear trend line through the yearly results in the Long Term dataset, providing an estimate of the rate of score changes over time. Positive or negative values show the direction of change and the coefficient the estimated annual rate of change, bolded values indicating a high probability (p. < .05) that the rate of change is different from zero. This form of trend estimation draws on the approach adopted to analyze national trends in PISA data (OECD, 2019a: Annex A7). The final column describes score trends using one of the nine PISA trajectory descriptors (OECD, 2019a: 133) used in the main text. Trajectories were decided with reference to the trend indicators shown and line plots of longer-term assessment results.

The table summarizes results and score trends for 36 different provincial math assessments. [20] Seven of these coincide with grade levels at which math performance is measured in the international and national assessments reviewed in the main text. Three

^[20] This is not intended as a definitive inventory as several culminating exams with low numbers of candidates in some provinces have been omitted in favour of exams with more participants. Several progress assessments that were discontinued during the years considered are also not included.

of these are not comparable, as British Columbia, New Brunswick, and Nova Scotia did not participate in the Grade 4 TIMSS assessments. Of the remaining four sets of comparable data points, Nova Scotia's Grade 8 and 10 PLANS assessments coincide with the PCAP and PISA assessments respectively, and British Columbia's now discontinued Grade 10 math exams and New Brunswick's new Grade 10 math assessment also coincide with PISA.

Trends for these directly comparable measures do not agree. Nova Scotia's Grade 8 PCAP results followed a steadily positive trajectory, while results from the province's internal assessments were classified as flat. Results from Nova Scotia's and British Columbia's Grade 10 exams are classified as flat, while both provinces' PISA trajectories were negative. The two Grade 10 results available for New Brunswick do not allow for trend estimates to compare with the province's steadily negative PISA trajectory.

The grades in table 5 divide into 13 Elementary level assessments (Grades 3 to 6), 8 Junior/ Intermediate level assessments (Grades 7 to 9), and 15 Secondary level assessments (Grades 10 to 12). All the secondary level assessments are cumulative examinations. Discounting the Grade 10 New Brunswick assessment for which there is currently insufficient data to estimate trends, results from 7 of the admissible 14 secondary level assessments have flat score trajectories (BC, MB, NL, NS, SKx3). In the remaining seven secondary level assessments, there are three positive flattening trajectories (AB, PE, QC), three steadily positive (PE, QCx2) and one U-shaped (AB). The U shaped and positive trajectories are concentrated in Alberta, Prince Edward Island, and Quebec. Alberta and Quebec were among the three highest scoring provinces in the 2018 PISA results, with Quebec outperforming all provinces. This is supported by the positive internal trends for all three of the Quebec secondary level assessments. Ontario, the second highest scoring province on PISA18, does not have comparable secondary level provincial results.

Three provinces with Junior/Intermediate grade assessments were classified as having negative score trajectories (AB, BC, NL), two positive trajectories (ONx2, MB) and two flat trajectories (NS, PE). As discussed in the main text, the Ontario results are weighted averages for students in Academic and Applied courses in both English and French language schools. Both of these Ontario Grade 9 results have statistically significant positive regression coefficients, as do Manitoba's Grade 7 numeracy scores. Manitoba is the only province with a positive Junior/Intermediate internal score trajectory and a positive PCAP Grade 8 trajectory.

Manitoba is also the only province with a positive internal score trajectory at the Elementary level (Grade 3). Alberta and British Columbia had flat score trajectories for Grade 6 and 4 respectively. The remaining four provinces with admissible data assess math performance at two grades within the Elementary level. Nova Scotia has flat score trajectories for both Grades 4 and 6, and Newfoundland and Labrador a flat trajectory for Grade 3 and a hump-shaped trajectory for Grade 6, while Prince Edward Island has an increasingly positive trajectory in Grade 3 and a flat trajectory in Grade 6. Ontario's Grade 3 and 6 math assessments both have statistically significant negative score trajectories. Ontario's Grade 4 TIMSS trajectory was classified as flat, with small score variations over the 12-year period considered (table 3, figure 4).

Table A1: Overview of provincial mathematics assessment programs with selected trend indicators

	Criterion	Assessment year		Score difference			Trajectory			
Alberta		2011	2016	2019	LT%∆	ST%∆	LTβ			
Grade 6 PAT	% acceptable standard	81.2	76.7	79.8	-1.7%	4.0%	-0.352	Flat		
Grade 9 PAT		73.9	75.5	67.4	-8.8%	-10.7%	-0.793	Increasingly negative		
Grade 12 Math 30-1 exam		(2013) 80.9	70.7	77.8	-3.8%	10.0%	-0.247	U shaped		
Grade 12 Math 30-2 exam		(2013) 69.5	75.4	76.5	10.1%	1.5%	0.561	Positive flattening		
British Columbia		2008	2014	2017	LT%∆	ST%∆	LTβ			
Grade 4 FSA	% meeting or exceeding expectations	76.7	78.9	77.4	0.9%	-1.9%	0.002	Flat		
Grade 7 FSA		77.3	74.5	72.2	-6.6%	-3.1%	-0.004	Negative		
Grade 10 FMP C10	Average mark	(2011) 67.7	(2013) 67.5	(2016) 67.8	0.1%	0.4%	0.033	Flat		
Manitoba		2010	2015	2018	LT%∆	ST%∆	LTβ			
Grade 3 Numeracy		29.1	34.4	40.5	39.2%	17.7%	1.022	Steadily positive		
Grade 7 Numeracy	% correct	32.2	37.8	40.9	27.0%	8.2%	1.166	Positive flattening		
Grade 12 Provincial exam avg.		62.5	61.4	59.9	-4.2%	-2.4%	-0.374	Flat		
New Brunswick (English)			2018	2019	LT%∆	ST%∆	LTβ			
Grade 4 PMA	% appropriate level or higher	na	na	62.3						
Grade 6 PMA		na	na	57.8	Insufficient data					
Grade 10 PMA		na	64.5	62.8						
Newfoundland & Labrador		2008	2013	2016	LT%∆	ST%∆	LTβ			
Grade 3		75.6	74.8	79.1	4.6%	5.7%	0.234	Flat		
Grade 6	% correct	64.6	66.9	63.1	-2.3%	-5.7%	-0.152	Hump shaped		
Grade 9		(2010) 66.4	68.2	67.2	1.2%	-1.5%	0.186	Increasingly negative		
Gr12 M3200 exam		80	78	77	-3.8%	-1.3%	-0.233	Flat		
Nova Scotia		2014	2016*	2019	LT%∆	ST%∆	LTβ			
Grade 4 PLANS	% at or above expectations	74	76	(2017) 77	4.1%	1.3%	0.9	Flat		
Grade 6 PLANS		73	68	71	-2.7%	4.4%	-0.143	Flat		
Grade 8 PLANS		57	62	57	0.0%	-8.1%	-0.049	Flat		
Grade 10 Exam	% correct	(2015) 67	71	70	4.5%	-1.4%	0.6	Flat		
Ontario		2008	2016	2019	LT%∆	ST%∆	LTβ			
Grade 3 (Eng & Fr)	% meeting or exceeding provincial standard	68	64	59	-13.2%	-7.8%	-0.009	Increasingly negative		
Grade 6 (Eng & Fr)		62	52	50	-19.4%	-3.8%	-0.013	Negative flattening		
Grade 9 Academic (Eng & Fr)		75	83	84	12.0%	1.2%	0.007	Positive flattening		
Grade 9 Applied (Eng & Fr)		34	45	43	26.5%	-4.4%	0.008	Positive flattening		

See next page for rest of table A1 and notes.

Table A1 continued

	Criterion	Assessment year		Score difference			Trajectory	
Prince Edward Island		2013	2016	2019	LT%∆	ST%∆	LTβ	
Grade 3	% meeting expectations	65	61	76	16.9%	24.6%	-0.4	Increasingly positive
Grade 6		56	77	73	30.4%	-5.2%	2.429	Flat
Grade 9		64	69	69	7.8%	0.0%	1.75	Flat
Grade 11 Academic	Average score	(2015) 55	60	60.5	10.0%	0.8%	1.55	Positive flattening
Grade 11 Pre Calculus		(2015) 60.5	66	74.5	23.1%	12.9%	3.55	Steadily positive
Quebec (Ministerial exam)		2012	2015	2018	LT%∆	ST%Δ	LTβ	
Math 3414 (Fr & Eng)	Average score	61.5	62.8	69.1	12.4%	10.0%	1.247	Steadily positive
Math 5426 (Fr & Eng)		74.5	79.2	78	4.7%	-1.5%	0.461	Positive flattening
Math 4426 (Fr & Eng)		65.8	70.4	77	17.0%	9.4%	1.554	Steadily positive
Saskatchewan (G12 exams)		2013	2016	2019	LT%∆	ST%∆	LTβ	
Work & apprentice 30	Department assigned scaled exam mark	69.8	69.7	68.9	-1.3%	-1.1%	0.066	Flat
Math Foundations 30		67.7	68.6	67.9	0.3%	-1.0%	0	Flat
Pre-Calculus 30		69.4	69.1	69.1	-0.4%	0.0%	-0.048	Flat

Notes: * 2016 designated baseline year in the new Program of Learning Assessment for Nova Scotia. Years shown are the latest in the academic pair, so that 2018 corresponds to 2017/18. Years in brackets before values in cells override the year shown in the column heading. Bolded Beta weights *p.*<.05

Source: Names and data collected from provincial websites.

About the Authors

Derek J. Allison

Derek J. Allison, B.Ed., M.Ed., Ph.D., is a Professor Emeritus in the Faculty of Education at the University of Western Ontario. Derek began his teaching career in England, before moving to Alberta, where he was a school principal. After completing his graduate work at the University of Alberta, he accepted a position with the faculty of education at the University of Western Ontario, where he taught social and legal foundations of education for 36 years, and skillfully guided hundreds of graduate students through advanced



research and study. He gained acclaim for his teaching, especially his outstanding lectures, and his skill as a mentor and advisor to graduate students. He has an extensive record in research and publication with particular interests in the organization and operation of schools, theories of leadership, and the philosophy of inquiry. He is the recipient of 10 teaching awards and the Distinguished Service Award of the Canadian Association for the Study of Educational Administration.

Vincent Geloso

Vincent Geloso, Fraser Institute Senior Fellow, is an Assistant Professor of Economics at King's University College, Western University, Canada and earned his PhD from the London School of Economics. Previously, he was postdoctoral fellow at Texas Tech University and earned his undergraduate degree from the University of Montreal. Professor Geloso specializes in the measurement of living standards today and in the distant past. He combines his specialization in economic history with a specializa-



tion in political economy in order to explain differences in living standards over time and space. His articles have been published in *Economics & Human Biology*, *Canadian Journal of Economics*, *Social Science Quarterly*, *Journal of Economic History*, *Health Policy & Planning* and *Historical Methods*. He has also authored opinion articles in the *Journal de Montréal*, *Journal de Québec*, *National Post*, *The Gazette*, *Le Devoir*, *La Presse*, *Le Soleil* and *Huffington Post Québec*.

Acknowledgments

The Fraser Institute thanks the Donner Canadian Foundation and the Lotte and John Hecht Memorial Foundation for their generous support of this publication. The Institute also wishes to thank the Weston Family Foundation; the foundation of the Institute's current research on education policy was built thanks to a very generous grant for an initiative in the name of Barbara Mitchell from the Weston Family Foundation. The authors wish to thank the unidentified reviewers for their helpful comments on an earlier draft. Any remaining errors are the sole responsibility of the authors. As the researchers have worked independently, the views and conclusions expressed in this paper do not necessarily reflect those of the Board of Directors of the Fraser Institute, the staff, or supporters.

Publishing Information

Distribution

These publications are available from http://www.fraserinstitute.org in Portable Document Format (PDF) and can be read with Adobe Acrobat® or Adobe Reader®, versions 7 or later. Adobe Acrobat Reader® DC, the most recent version, is available free of charge from Adobe Systems Inc. at <get.adobe.com/reader/>. Readers having trouble viewing or printing our PDF files using applications from other manufacturers (e.g., Apple's Preview) should use Reader® or Acrobat®.

Ordering publications

To order printed publications from the Fraser Institute, please contact us via e-mail: sales@fraserinstitute.org; telephone: 604.688.0221, ext. 580 or, toll free, 1.800.665.3558, ext. 580; or fax: 604.688.8539.

Media

For media enquiries, please contact our communications department via e-mail: communications@fraserinstitute.org; telephone: 604.714.4582.

Copyright

Copyright © 2021 by the Fraser Institute. All rights reserved. No part of this publication may be reproduced in any manner whatsoever without written permission except in the case of brief passages quoted in critical articles and reviews.

ISBN

978-0-88975-635-9.

Citation

Allison, Derek J., and Vincent Geloso (2021). *Math Performance in Canada*. Fraser Institute. http://www.fraserinstitute.org.

Supporting the Fraser Institute

To learn how to support the Fraser Institute, please contact us via post: Development Department, Fraser Institute, Fourth Floor, 1770 Burrard Street, Vancouver, British Columbia, V6J 3G7, Canada; telephone: toll-free to 1.800.665.3558, ext. 548; e-mail: development@fraserinstitute.org; or visit our webpage: <www.fraserinstitute.org/support-us/overview.aspx>.

Purpose, Funding, and Independence

The Fraser Institute provides a useful public service. We report objective information about the economic and social effects of current public policies, and we offer evidence-based research and education about policy options that can improve the quality of life.

The Institute is a non-profit organization. Our activities are funded by charitable donations, unrestricted grants, ticket sales, and sponsorships from events, the licensing of products for public distribution, and the sale of publications.

All research is subject to rigorous review by external experts, and is conducted and published separately from the Institute's Board of Directors and its donors.

The opinions expressed by authors are their own, and do not necessarily reflect those of the Institute, its Board of Directors, its donors and supporters, or its staff. This publication in no way implies that the Fraser Institute, its directors, or staff are in favour of, or oppose the passage of, any bill; or that they support or oppose any particular political party or candidate.

As a healthy part of public discussion among fellow citizens who desire to improve the lives of people through better public policy, the Institute welcomes evidence-focused scrutiny of the research we publish, including verification of data sources, replication of analytical methods, and intelligent debate about the practical effects of policy recommendations.

About the Fraser Institute

Our mission is to improve the quality of life for Canadians, their families and future generations by studying, measuring and broadly communicating the effects of government policies, entrepreneurship and choice on their well-being.

Notre mission consiste à améliorer la qualité de vie des Canadiens et des générations à venir en étudiant, en mesurant et en diffusant les effets des politiques gouvernementales, de l'entrepreneuriat et des choix sur leur bien-être.

Peer review—validating the accuracy of our research

The Fraser Institute maintains a rigorous peer review process for its research. New research, major research projects, and substantively modified research conducted by the Fraser Institute are reviewed by experts with a recognized expertise in the topic area being addressed. Whenever possible, external review is a blind process. Updates to previously reviewed research or new editions of previously reviewed research are not reviewed unless the update includes substantive or material changes in the methodology.

The review process is overseen by the directors of the Institute's research departments who are responsible for ensuring all research published by the Institute passes through the appropriate peer review. If a dispute about the recommendations of the reviewers should arise during the Institute's peer review process, the Institute has an Editorial Advisory Board, a panel of scholars from Canada, the United States, and Europe to whom it can turn for help in resolving the dispute.

Editorial Advisory Board

Members

Prof. Terry L. Anderson Prof. Herbert G. Grubel

Prof. Robert Barro Prof. James Gwartney

Prof. Jean-Pierre Centi Prof. Ronald W. Jones

Prof. John Chant Dr. Jerry Jordan

Prof. Bev Dahlby Prof. Ross McKitrick

Prof. Erwin Diewert Prof. Michael Parkin

Prof. Stephen Easton Prof. Friedrich Schneider

Prof. J.C. Herbert Emery Prof. Lawrence B. Smith

Prof. Jack L. Granatstein Dr. Vito Tanzi

Past members

Prof. Armen Alchian* Prof. F.G. Pennance*

Prof. Michael Bliss* Prof. George Stigler*†

Prof. James M. Buchanan*† Sir Alan Walters*

Prof. Friedrich A. Hayek*† Prof. Edwin G. West*

Prof. H.G. Johnson*

^{*} deceased; † Nobel Laureate